Xth HAND WRITTEN MATERIALS

ALL CHAPTER EXAMPLES SUMS

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CHAPTER – 1 (Examples)

RELATIONS AND FUNCTIONS

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CHAPTER - 1

RELATIONS AND FUNCTIONS

Example 1.1

If $A = \{1, 3, 5\}$ and $B = \{2, 3\}$, then (i) find $A \times B$ and $B \times A$. (ii) Is $A \times B = B \times A$? If not why ? (iii) Show that $n(A \times B) = n(B \times A) = n(A) \times n(B)$.

Solution:

Given that $A = \{1, 3, 5\}$ and $B = \{2, 3\}$

(i)

$A \times B = \{1, 3, 5\} \times \{2, 3\}$

$= \{(1, 2), (1, 3), (3, 2), (3, 3), (5, 2), (5, 3)\} \longrightarrow (1)$

$B \times A = \{2, 3\} \times \{1, 3, 5\}$

$= \{(2, 1), (2, 3), (3, 1), (3, 3), (2, 5), (3, 5)\} \longrightarrow (2)$

(ii) From (1) & (2)

$A \times B \neq B \times A$ as $(1, 2) \neq (2, 1)$ and $(1, 3) \neq (3, 1)$, etc.

(iii) $n(A) = 3$, $n(B) = 2$.

From (1) & (2)

$n(A \times B) = n(B \times A) = 6$

$n(A) \times n(B) = 3 \times 2 = 6$

$n(B) \times n(A) = 2 \times 3 = 6$

$n(A \times B) = n(B \times A) = n(A) \times n(B) = 6$

$\therefore \quad n(A \times B) = n(B \times A) = n(A) \times n(B)$

Example 1.2

If $A \times B = \{(3, 2), (3, 4), (5, 2), (5, 4)\}$ then find A and B.

Solution

$A \times B = \{(3, 2), (3, 4), (5, 2), (5, 4)\}$

$A = \{$ Set of all first coordinates of elements of $A \times B\}$

$\therefore A = \{3, 5\}$

$B = \{$ Set of all second coordinates of elements of $A \times B\}$

$\therefore B = \{2, 4\}$

$\therefore A = \{3, 5\}$ and $B = \{2, 4\}$

Example 1.3

Let, $A = \{x \in N \mid 1 < x < 4\}$; $B = \{x \in W \mid 0 \leq x < 2\}$ and $C = \{x \in N \mid x < 3\}$ then verify that.

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Sol

$A = \{x \in N \mid 1 < x < 4\}$; $B = \{x \in W \mid 0 \leq x < 2\}$

$C = \{x \in N \mid x < 3\}$

LHS: $A \times (B \cup C)$

~~$B \cup C = \{0$~~

$A = \{2, 3\}$, $B = \{0, 1\}$, $C = \{1, 2\}$

$B \cup C = \{0, 1\} \cup \{1, 2\} = \{0, 1, 2\}$

$A \times (B \cup C) = \{2, 3\} \times \{0, 1, 2\}$

$A \times (B \cup C) = \{(2,0), (2,1), (2,2), (3,0), (3,1), (3,2)\} \longrightarrow (1)$

RHS: $(A \times B) \cup (A \times C)$

$A \times B = \{2, 3\} \times \{0, 1\} = \{(2,0), (2,1), (3,0), (3,1)\}$

$A \times C = \{2, 3\} \times \{1, 2\} = \{(2,1), (2,2), (3,1), (3,2)\}$

$(A \times B) \cup (A \times C) = \{(2,0), (2,1), (3,0), (3,1)\} \cup \{(2,1), (2,2), (3,1), (3,2)\}$

$(A \times B) \cup (A \times C) = \{(2,0), (2,1), (2,2), (3,0), (3,1), (3,2)\} \longrightarrow (2)$

From (1) & (2)

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

Hence verified.

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Sol

Let $A = \{2, 3\}$, $B = \{0, 1\}$, $C = \{1, 2\}$

LHS: $A \times (B \cap C)$

$B \cap C = \{0, 1\} \cap \{1, 2\} = \{1\}$

$A \times (B \cap C) = \{2,3\} \times \{1\} = \{(2,1), (3,1)\}$ ③

$\longrightarrow$ ①

RHS: $(A \times B) \cap (A \times C)$

$A \times B = \{2,3\} \times \{0,1\} = \{(2,0), (3,1), (2,1), (3,0)\}$

$A \times C = \{2,3\} \times \{1,2\} = \{(2,1), (2,2), (3,1), (3,2)\}$

$(A \times B) \cap (A \times C) = \{(2,0), (3,1), (2,1), (3,0)\} \cap \{(2,1), (2,2), (3,1), (3,2)\}$

$(A \times B) \cap (A \times C) = \{(2,1), (3,1)\}$ $\longrightarrow$ ②

From ① & ②

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

Hence verified.

Example 1.4:

Let $A = \{3, 4, 7, 8\}$ and $B = \{1, 7, 10\}$. Which of the following sets are relations from A to B?

(i) $R_1 = \{(3,7), (4,7), (7,10), (8,1)\}$

(ii) $R_2 = \{(3,1), (4,12)\}$

(iii) $R_3 = \{(3,7), (4,10), (7,7), (7,8), (8,11), (8,7), (8,10)\}$

Sol

$A \times B = \{(3,1), (3,7), (3,10), (4,1), (4,7), (4,10), (7,1), (7,10), (7,7), (8,1), (8,7), (8,10)\}$

(i) $R_1 \subseteq A \times B$, Thus R_1 is a relation from A to B

(ii) Here, $(4,12) \in R_2$, But $(4,12) \notin A \times B$.

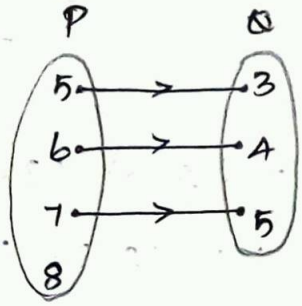
$\therefore$ R_2 is not a relation from A to B.

(iii) Here, $(7,8) \in R_3$, but $(7,8) \in A \times B$.

$\therefore$ R_3 is not a relation from A to B.

Example 1.5:

The Arrow diagram shows in fig a relationship between the sets P and Q. Write the relation in (i) set builder form. (ii) Roaster form, (iii) What is domain and range of R?



Sol

(i) Set builder form of $R = \{(x, y) \mid y = x - 2, x \in P, y \in Q\}$

(ii) Roster form $R = \{(5,3), (6,4), (7,5)\}$

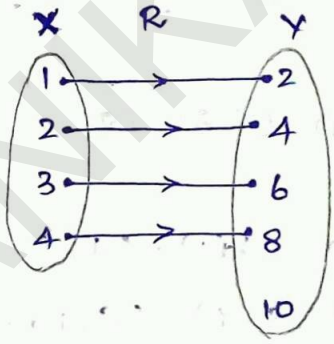
(iii) Domain of $R = \{5, 6, 7\}$

Range of $R = \{3, 4, 5\}$

Example 1.6:

Let $X = \{1, 2, 3, 4\}$ and $Y = \{2, 4, 6, 8, 10\}$ and $R = \{(1,2), (2,4), (3,6), (4,8)\}$. Show that R is a function and find its domain, co-domain and Range?

Sol



Pictorial representation of R is given. From the diagram, Each $x \in X$, there exists only one $y \in Y$ thus all elements in X have only one image in Y.

$\therefore$ R is a function.

Domain $X = \{1, 2, 3, 4\}$

Co-domain $Y = \{2, 4, 6, 8, 10\}$

Range of $f = \{2, 4, 6, 8\}$

Example 1.7:

A relation $f: X \to Y$ is defined by $f(x) = x^2 - 2$, where $X = \{-2, -1, 0, 3\}$, $Y = \mathbb{R}$. (i) List of the elements of f?

(ii) Is f is a function?

Sol

$f(x) = x^2 - 2$, where $X = \{-2, -1, 0, 3\}$.

(i) $f(-2) = (-2)^2 - 2 = 4 - 2 = 2$

$f(-1) = (-1)^2 - 2 = 1 - 2 = -1$

$f(0) = (0)^2 - 2 = 0 - 2 = -2$

$f(3) = (3)^2 - 2 = 9 - 2 = 7$

$\therefore f = \{(-2, 2), (-1, -1), (0, -2), (3, 7)\}$

(ii) Each Element in the domain of f has a unique image.

$\therefore f$ is a function.

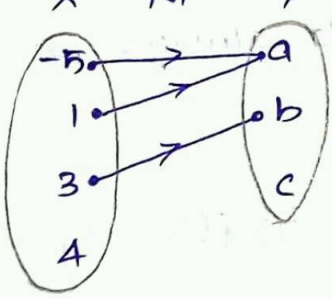
Example 1.8:

If $X = \{-5, 1, 3, 4\}$ and $Y = \{a, b, c\}$, then which of the following relations are function from X to Y?

(i) $R_1 = \{(-5, a), (1, a), (3, b)\}$

Sol

Arrow diagram of R_1

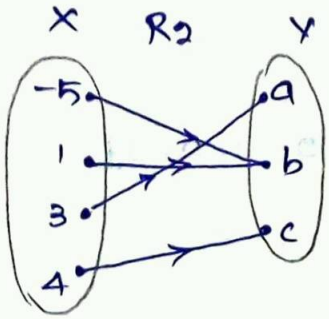


R_1 is not function as $4 \in X$, does not have an image in Y.

(ii) $R_2 = \{(-5, b), (1, b), (3, a), (4, c)\}$

Sol

Arrow diagram of R_2

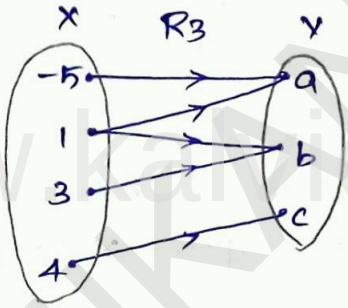


R_2 is a function as each element of X has an unique image in Y.

(iii) $R_3 = \{(-5, a), (1, a), (3, b), (4, c), (1, b)\}$

Sol

Arrow diagram of R_3



R_3 is not a function as $1 \in X$ has two images $a \in Y$ and $b \in Y$.

The image of an element should always be unique.

Example 1.9:

Given $f(x) = 2x - x^2$, find (i) $f(1)$, (ii) $f(x+1)$, (iii) $f(x) + f(1)$.

Sol

(i) $x = 1$, $f(x) = 2x - x^2$, $f(1)$

$f(1) = 2(1) - (1)^2$

$= 2 - 1$

$f(1) = 1$

(ii) $x = x + 1$

$f(x+1) = 2(x+1) - (x+1)^2$

$= 2x + 2 - (x^2 + 2x + 1)$

$= 2x + 2 - x^2 - 2x - 1$

$f(x+1) = -x^2+1$

(iii) $f(x) + f(1)$

$f(x) = 2x - x^2$

$f(1) = 1$

$\Rightarrow (2x - x^2) + 1$

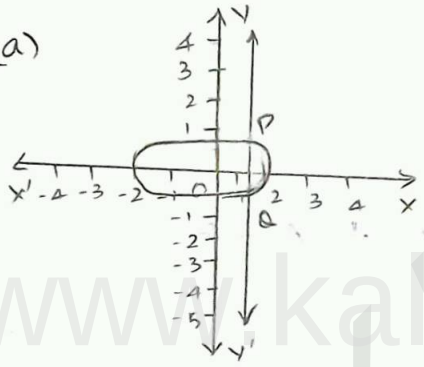
$= 2x - x^2 + 1$

$f(x)+f(1) = -x^2 + 2x + 1$

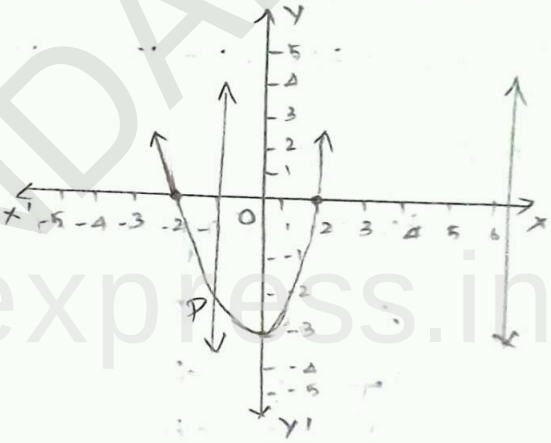
Example 1.10:

Using vertical line test, determine which of the following curves represent a function

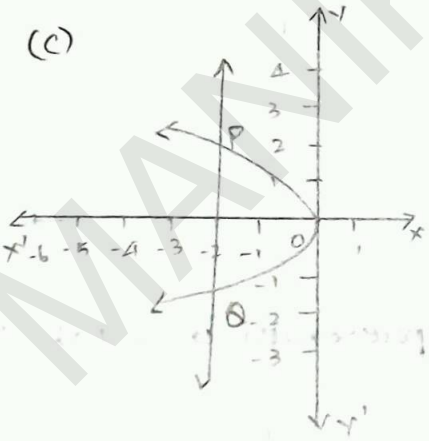
(a)



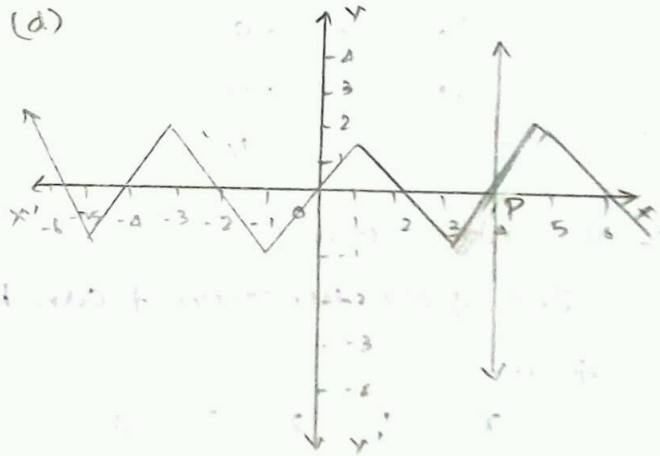
(b)



(c)



(d)



Sol

(i) The curves in (a) and (c) do not represent a function as the verticle lines meet the curves in two points P and Q.

(ii) The curves in (b) and (d) represent a function as the vertical lines meet the curves in at most one point.

Example 1.11 :

Let $A = \{1, 2, 3, 4\}$, $B = \{2, 5, 8, 11, 14\}$ be two sets.

Let $f : A \to B$ be a function given by $f(x) = 3x - 1$.

Represent this function.

(i) by Arrow diagram (ii) in a table form.

(iii) As a set of ordered pairs (iv) in a graphical form

Sol

$A = \{1, 2, 3, 4\}$; $B = \{2, 5, 8, 11, 14\}$: $f(x) = 3x - 1$

$f(1) = 3(1) - 1 = 3 - 1 = 2$

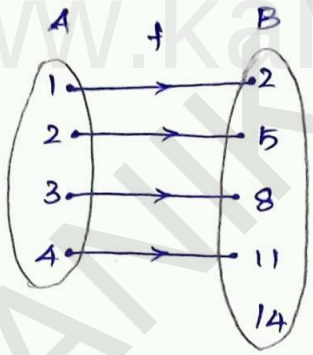
$f(2) = 3(2) - 1 = 6 - 1 = 5$

$f(3) = 3(3) - 1 = 9 - 1 = 8$

$f(4) = 3(4) - 1 = 12 - 1 = 11$

(i) Arrow diagram :

Arrow diagram of function $f : A \to B$



(ii) Table form :

The given function f can be represented in a tabular form.

x	1	2	3	4
$f(x)$	2	5	8	11

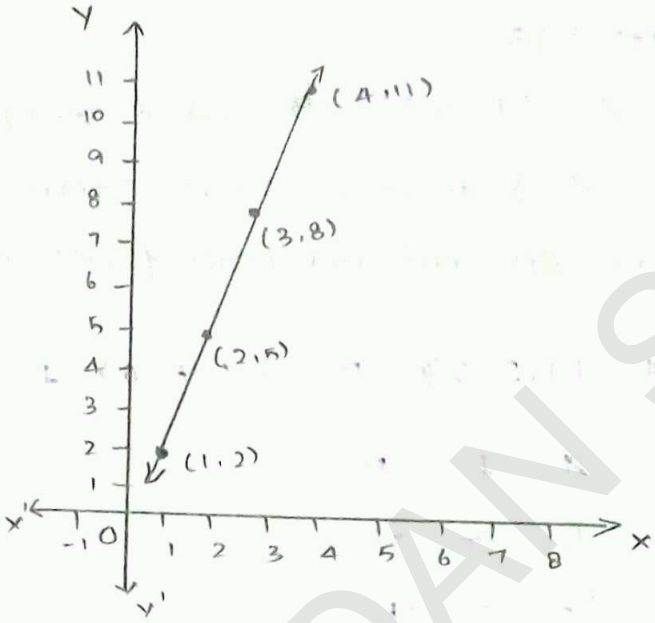
(iii) Set of ordered pairs.

The function f can be represented as a set of ordered pairs as

$f = \{(1, 2), (2, 5), (3, 8), (4, 11)\}$

(iv) Graphical form:

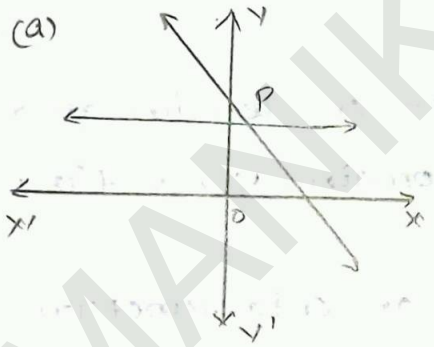
In the adjacent XY - plane the points (1, 2), (2, 5), (3, 8), (4, 11) are plotted.



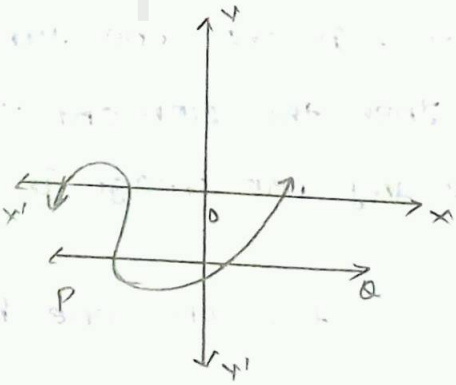
Example 1.12:

Using horizontal line test, determine which of the following functions are one - one.

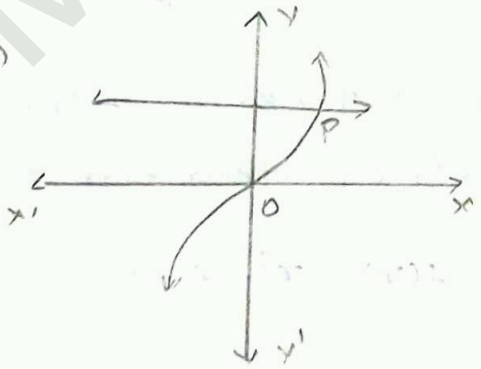
(a)



(b)



(c)



Sol.

The curves in (a) and (b) represent a one - one function as the horizontal lines meet the curves in only one point P.

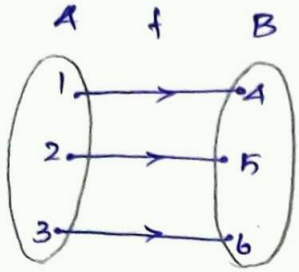
The curve in (b) does not represent a one-one function since the horizontal line intersect the curve at two points P and Q.

Example 1.13:

Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $f = \{(1,4), (2,5), (3,6)\}$ be a function from A to B. Show that f is one-one but not onto function.

Sol

$A = \{1, 2, 3\}$, $B = \{4, 5, 6\}$; $f = \{(1,4), (2,5), (3,6)\}$



f is a function from A to B and for different elements in A, there are different images in B. Hence f is one-one function.

Then the element 7 in the co-domain does not have any pre image in the domain. Hence f is not onto.

$\therefore$ f is one-one but not an onto function.

Example 1.14:

If $A = \{-2, -1, 0, 1, 2\}$ and $f: A \to B$ is an onto function defined by $f(x) = x^2 + x + 1$ then find B.

Sol

$A = \{-2, -1, 0, 1, 2\}$ and $f(x) = x^2 + x + 1$

$f(-2) = (-2)^2 + (-2) + 1$

$f(-2) = 4 - 2 + 1 = 3$

$f(-1) = (-1)^2 + (-1) + 1$

$f(-1) = 1 - 1 + 1 = 1$

$f(0) = (0)^2 + 0 + 1$

$f(0) = 1$

$f(1) = (1)^2 + 1 + 1$

$f(1) = 1 + 1 + 1 = 3$

$f(2) = (2)^2 + 2 + 1$

$f(2) = 4 + 2 + 1 = 7$

$\therefore B = \{1, 3, 7\}$

Example 1.15:

Let f be a function $f: N \to N$ be defined by $f(x) = 3x + 2$ $x \in N$

(i) Find the image of 1, 2, 3

(ii) Find the pre-image of 29, 53

(iii) Identify the type of function.

Sol

The function $f: N \to N$ is defined by $f(x) = 3x + 2$

(i) If $x = 1$; $f(1) = 3(1) + 2 = 5$

If $x = 2$; $f(2) = 3(2) + 2 = 8$

If $x = 3$; $f(3) = 3(3) + 2 = 11$

The images of 1, 2, 3 are 5, 8, 11

(ii) If x is the pre-image of 29, then $f(x) = 29$,

$3x + 2 = 29$

$3x = 29 - 2$

$3x = 27$

$x = 27/3 = 9$

If x is the pre-image of 53, then $f(x) = 53$

$3x + 2 = 53$

$3x = 53 - 2$

$3x = 51$

$x = 51/3 = 17$

$\therefore$ The pre-image of 29 and 53 are 9 and 17.

(iii) Since different elements of N have different images in the co-domain, the function f is one-one function

The co-domain of f is N.

But the range of $f = \{5, 8, 11, 14, 17, \ldots\}$ is a proper subset of N.

$\therefore$ f is not an onto function. that is f is an into function.

$\therefore$ f is one-one and into function.

Example 1.16:

Forensic scientists can determine the height (in cm) of a person based on the length of the thigh bone. they usually do so using the function $h(b) = 2.47b + 54.10$, where b is the length of the thigh bone.

(i) verify the function h is one-one or not

(ii) Also find the height of a person if the length of his thigh bone is 50 cm.

(iii) Find the length of the thigh bone if the height of a person is 147.96 cm.

Sol

(i) To check if h is one-one, we assume that $h(b_1) = h(b_2)$.

$$2.47b_1 + 54.10 = 2.47b_2 + 54.10$$

$$2.47b_1 = 2.47b_2 + 54.10 - 54.10$$

$$2.47b_1 = 2.47b_2$$

$$b_1 = \frac{2.47}{2.47} b_2$$

$$b_1 = (1) b_2$$

$$b_1 = b_2.$$

$$h(b_1) = h(b_2) \Rightarrow b_1 = b_2$$

$\therefore$ the function h is one-one.

(ii) If the length of the thigh bone $b = 50$, then the height is

$h(50) = (2.47 \times 50) + 54.10$

$= 123.5 + 54.10$

$h(50) = 177.6$ cm.

(iii) If the height of a person is 147.96 cm. then

$h(b) = 147.96$

and so the length of the thigh bone is given by

$2.47b + 54.10 = 147.96$

$2.47b = 147.96 - 54.10$

$2.47b = 93.86$

$b = \frac{93.86}{2.47} = 38$

$\therefore$ The length of the thigh bone is 38 cm.

Example 1.17

Let f be a function from R to R defined by $f(x) = 3x - 5$. Find the values of a and b given that $(a, 4)$ and $(1, b)$ belong to f.

Sol

$f(x) = 3x - 5$ can be written as $f = \{(x, 3x-5) \mid x \in R\}$

$(a, 4)$ means the image of a is 4 (ie) $f(a) = 4$

$f(a) = 3(a) - 5$

$4 = 3a - 5$

$3a = 5 + 4$

$3a = 9$

$a = 9/3 \Rightarrow 3$

$(1, b)$ means the image of 1 is b (i) $f(1) = b$

$f(x) = 3x - 5$

$f(1) = 3(1) - 5$

$b = 3 - 5$

$\boxed{b = -2}$

Example 1.18:

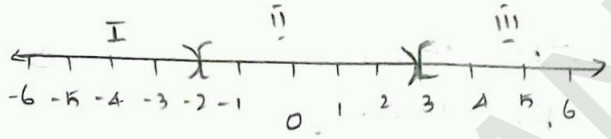
If the function $f: R \to R$ is defined by

$$f(x) = \begin{cases} 2x+7; & x < -2 \\ x^2-2; & -2 \le x < 3 \\ 3x-2; & x \ge 3 \end{cases}$$; then find the values of

(i) $f(4)$ (ii) $f(-2)$ (iii) $f(4)+2f(1)$ (iv) $\dfrac{f(1)-3f(4)}{f(-3)}$

Sol

The function f is defined by three values in interval I, II, III as



For a given value of $x = a$, find out the interval at which the point a is located, there also find $f(a)$ using the particular value defined in the interval

(i) $x = 4$ lie in the third interval

$\therefore$ $f(x) = 3x - 2$

$f(4) = 3(4) - 2 = 10$

(ii) $x = -2$ lies in the second interval

$f(x) = x^2 - 2$

$f(-2) = (-2)^2 - 2 = 4 - 2 = 2$

(iii) From (i) $f(4) = 10$

to find $f(1)$, $x = 1$ lies in the second interval

$f(x) = x^2 - 2$

$f(1) = (1)^2 - 2 = 1 - 2 = -1$

$f(4) + 2f(1) = 10 + 2(-1) = 10 - 2 = 8$.

(iv) $f(4) = 10$; $f(1) = -1$

to find $f(-3)$, $x = -3$ lies in the first interval

$f(x) = 2x + 7$

$f(-3) = 2(-3) + 7$

$= -6 + 7 = 1$

$$\frac{f(1)-3f(4)}{f(-3)} = \frac{-1-3(10)}{1} = \frac{-1-30}{1} = \frac{-31}{1} = -31$$

Example 1.19

Find $f \circ g$ and $g \circ f$ when $f(x) = 2x+1$ and $g(x) = x^2-2$

Sol

$f(x) = 2x+1$, $g(x) = x^2-2$

$$f \circ g(x) = f(g(x)) = f(x^2-2) = 2(x^2-2)+1$$

$$= 2x^2-4+1$$

$$f \circ g(x) = 2x^2-3$$

$$g \circ f(x) = g(f(x)) = g(2x+1) = (2x+1)^2-2$$

$$= 4x^2+4x+1-2$$

$$g \circ f(x) = 4x^2+4x-1$$

$$\therefore f \circ g \neq g \circ f$$

Example 1.20

Represent the function $f(x) = \sqrt{2x^2-5x+3}$ as a composition of two functions

Sol

$f_2(x) = 2x^2-5x+3$ and $f_1 x = \sqrt{x}$

$$f(x) = \sqrt{2x^2-5x+3} = \sqrt{f_2(x)}$$

$$\sqrt{f_2(x)} = f_1(f_2(x))$$

$$= f_1 f_2(x)$$

Example 1.21

If $f(x) = 3x-2$, $g(x) = 2x+k$ and if $f \circ g = g \circ f$, then find the value of k.

Sol

$f(x) = 3x-2$, $g(x) = 2x+k$

$$f \circ g(x) = f(g(x)) = f(2x+k) = 3(2x+k)-2$$

$f \circ g(x) = 6x + 3k - 2$

$g \circ f(x) = g(f(x)) = g(3x-2) = 2(3x-2) + k$

$g \circ f(x) = 6x - 4 + k$

Given: $g \circ f = f \circ g$

$6x - 4 + k = 6x + 3k - 2$

$6x - 6x = 3k - k - 2 + 4$

$0 = 2k + 2$

$-2 = 2k$

$k = 2/-2 = -1$

$\boxed{k = -1}$

Example 1.22

Find k if $f \circ f(k) = 5$, where $f(k) = 2k - 1$

Sol

$f \circ f(k) = f(f(k))$

$= f(2k-1)$

$= 2(2k-1) - 1$

$f \circ f(k) = 4k - 2 - 1 \Rightarrow 4k - 3$

But $f \circ f(k) = 5$

$\therefore 5 = 4k - 3$

$5 + 3 = 4k$

$8 = 4k$

$\boxed{k = 2}$

Example 1.23

If $f(x) = 2x+3$, $g(x) = 1-2x$, $h(x) = 3x$, prove that $f \circ (g \circ h) = (f \circ g) \circ h$.

Sol

$f(x) = 2x+3$, $g(x) = 1-2x$, $h(x) = 3x$

Now,

LHS: $f \circ (g \circ h)$

$(g \circ h)(x) = g(h(x)) = g(3x) = 1-2(3x) = 1-6x$

$f \circ (g \circ h)x = f(g(h(x))) = f(1-6x)$

$= 2(1-6x)+3$

$= 2-12x+3$

$f \circ (g \circ h) = 5-12x \longrightarrow (1)$

RHS: $(f \circ g) \circ h$

$(f \circ g)(x) = f(g(x)) = f(1-2x) = 2(1-2x)+3$

$= 2-4x+3$

$f \circ g(x) = 5-4x$

$(f \circ g) \circ h = (f(g(x)))h(x) = h(x)(5-4x) = 5-4(3x)$

$(f \circ g) \circ h = 5-12x \longrightarrow (2)$

From (1) & (2)

$f \circ (g \circ h) = (f \circ g) \circ h$

Hence proved.

Example 1.24:

Find x if $gff(x) = fgg(x)$, given $f(x) = 3x+1$ and $g(x) = x+3$

Sol

$gff(x) = g[f\{f(x)\}]$ [∵ this mean "g of f of f of x"]

$= g[f[3x+1]]$

$= g[3(3x+1)+1]$

$= g[9x+3+1] = g[9x+4]$

$= (9x+4)+3$

$gff(x) = 9x+7 \longrightarrow (1)$

$fgg(x) = f[g\{g(x)\}]$ ["This mean f of g of g of x"]

$= f[g[x+3]]$

$= f[(x+3)+3]$

$= f[x+6]$

$= 3(x+6)+1$

$= 3x+18+1$

$fgg(x) = 3x+19 \longrightarrow (2)$

solving ① & ②, Two equation be equal

$$9x + 7 = 3x + 19$$

$$9x - 3x = 19 - 7$$

$$6x = 12$$

$$x = 12/6$$

$$\boxed{x = 2}$$

Xth HAND WRITTEN MATERIALS

CHAPTER – 2 (Examples)

NUMBERS AND SEQUENCES

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P.G Teacher Mathematics

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CHAPTER - 2

NUMBERS AND SEQUENCES

Example 2.1

We have 34 cakes. Each boxes can hold 5 cakes only. How many boxes we need to pack and how many cakes are unpacked.

Sol

6 Boxes are required to pack 30 cakes with 4 left cakes left over. This distribution of cakes can be understood as follows:

34	=	5	×	6	+	4
total number of cakes	=	Number of cakes in each box	×	Number of boxes	+	Number of cakes left over
↓		↓		↓		↓
(Dividend) (a)	=	(divisor) b	×	(Quotient) q	+	remainder (r)

Example 2.2.

Find the Quotient and remainder when a is divided by b in the following cases.

(i) $a = -12$, $b = 5$, (ii) $a = 17$, $b = -3$ (iii) $a = -19$, $b = -4$

Sol

(i) $a = -12$, $b = 5$

By Euclid's division lemma

$a = bq + r$, where $0 \le r < |b|$

$-12 = 5(-3) + 3$, where $0 \le r < |5|$

$\therefore$ quotient $q = -3$, remainder $r = 3$

(ii) $a = 17$, $b = -3$

By Euclid's division lemma

$a = bq + r$, where $0 \le r < |b|$

$17 = (-3)(-5) + 2$, $0 \le r < |-3|$

$\therefore$ Quotient $Q = -5$, remainder $r = 2$.

(iii) $a = -19$, $b = -4$

By Euclid's division lemma

$a = bq + r$, where $0 \le r < |b|$

$-19 = (-4) \times (5) + 1$, $0 \le r < |-4|$

$\therefore$ Quotient $q = 5$, remainder $r = 1$.

Example 2.3

Show that the square of an odd integer is of the form $4q+1$, for some integer q.

Sol

Let x be any integer. Since any odd integer is one more than an even integer, we have $x = 2k+1$, for some integers k.

$$x = 2k+1$$

$$x^2 = (2k+1)^2$$

$$= 4k^2 + 4k + 1$$

$$= 4k(k+1) + 1$$

$= 4q+1$, where $q = k(k+1)$ in some integer.

Example 2.4

If the highest common factor of 210 and 55 is expressible in the form $55x - 325$, find x.

Sol

Using Euclid's Division Algorithms, let us find the HCF of given numbers.

$$210 = 55 \times 3 + 45$$

$$55 = 45 \times 1 + 10$$

$$45 = 10 \times 4 + 5$$

$$10 = 5 \times 2 + 0$$

The remainder is zero.

So, the last divisor 5 is the Highest Common Factor (HCF) of 210 and 55

$\therefore$ HCF is expressible in the form

$$55x - 325 = 5$$

$$55x = 5 + 325$$

$$55x = 330$$

$$x = \frac{330}{55}$$

$$\boxed{x = 6}$$

Example 2.5:

Find the greatest number that will divide 445 and 572 leaving remainder 4 and 5 respectively.

Sol: Since the remainders are 4, 5 respectively. the required number is the HCF of the number

$$445 - 4 = 441, \quad 572 - 5 = 567$$

Hence, we will determine the HCF of 441 and 567 using Euclid's Division Algorithm,

$$567 = 441 \times 1 + 126$$

$$441 = 126 \times 3 + 63$$

$$126 = 63 \times 2 + 0$$

$\therefore$ HCF of 441, 567 = 63, So the required number is 63.

Example 2.6

Find the HCF of 396, 504, 636.

Sol: To find HCF of three given numbers, first we have to find HCF of the two numbers.

to find HCF of 396 and 504

using Euclid's division Algorithm,

$$504 = 396 \times 1 + 108$$

The remainder is $108 \neq 0$

Again using Euclid's division Algorithm.

$$396 = 108 \times 3 + 72$$

The remainder is $72 \neq 0$

Again using Euclid's division Algorithm.

$$108 = 72 \times 1 + 36$$

The remainder is $36 \neq 0$

Again using Euclid's division Algorithm.

$$72 = 36 \times 2 + 0$$

The remainder is zero.

$\therefore$ HCF of 396, 504 = 36

to find the HCF of 636 and 36 using Euclid's division Algorithm

$$636 = 36 \times 17 + 24$$

The remainder is $24 \neq 0$

Again using Euclids division Algorithm.

$$36 = 24 \times 1 + 12$$

The remainder is $12 \neq 0$

Again using Euclid's division Algorithm.

$$24 = 12 \times 2 + 0$$

The remainder is zero

$\therefore$ HCF of 636 and 36 = 12.

$\therefore$ Highest common factor of 396, 504 and 636 is 12.

Example 2.7

In the given factorisation, find the numbers m and n.

Sol value of the first box from bottom = 5×2
$= 10$

value of $n = 5 \times 10 = 50$

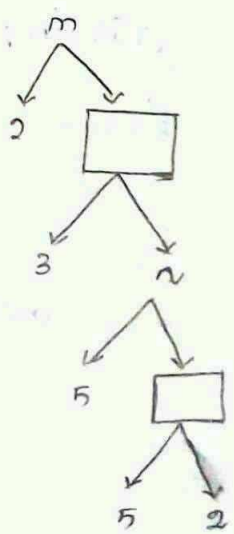
value of the second box from bottom.

$$3 \times 50 = 150$$

value of $m = 2 \times 150 = 300$

$\therefore$ The required numbers are

$m = 300$, $n = 50$



Example 2.8

Can the number 6^n, n being a natural number end with the digit 5? Give reason for your answer

Sol

Since $6^n = (2 \times 3)^n = 2^n \times 3^n$

2 is a factor of 6^n. So 6^n is always even.

But any number whose last digit is 5 is always odd.

Hence, 6^n cannot end with the digit 5.

Example 2.9

Is $7 \times 5 \times 3 \times 2 + 3$ a composite number? Justify your Answer.

Sol

Yes, the given number is a composite number, because.

$$7 \times 5 \times 3 \times 2 + 3 = 3 \times (7 \times 5 \times 2 + 1) = 3 \times 71.$$

Since, the given number can be factorized in terms of two primes, it is a composite number.

Example 2.10

'a' and 'b' are two positive integer such that $a^b \times b^a = 800$, Find 'a' and 'b'

Sol

The number 800 can be factorized as

$$800 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 = 2^5 \times 5^2$$

Hence $a^b \times b^a = 2^5 \times 5^2$

This implies that $a = 2, b = 5$ (or) $a = 5, b = 2$.

Example 2.11

Find the remainder when 70004 and 778 is divided by 7.

Sol

Since 70000 is divisible by 7

$$70000 \equiv 0 \pmod 7$$

$$70000 + 4 \equiv 0 + 4 \pmod 7$$

$$70004 \equiv 4 \pmod 7$$

$\therefore$ the remainder when 70004 is divisible by 7 is 4.

$\because$ 777 is divisible by 7

$$777 \equiv 0 \pmod 7$$

$$777 + 1 \equiv 0 + 1 \pmod 7$$

$$778 \equiv 1 \pmod 7$$

$\therefore$ the remainder when 778 is divisible by 7 is 1

Example 2.12

Determine the value of d such that $15 \equiv 3 \pmod d$.

Sol

$15 \equiv 3 \pmod d$ means $15 - 3 = kd$, for some integer k.

$$15 - 3 = kd$$

$$12 = kd$$

d divides 12.

The divisor of 12 are 1, 2, 3, 4, 6, 12. But d should be larger than 3.

So the possible value for d are 4, 6, 12.

Example 2.13

Find the least positive value of x such that.

(i) $67 + x \equiv 1 \pmod 4$ (ii) $98 \equiv (x+4) \pmod 5$

Sol

(i) $67 + x \equiv 1 \pmod 4$

$67 + x - 1 = 4n$, for some integer n

$66 + x = 4n$

$66 + x$ is a multiple of 4.

$\therefore$ The least positive value of x must be 2, since 68 is the nearest multiple of 4 more than 66.

(ii) $98 \equiv (x+4) \pmod 5$

$98 - (x+4) \equiv \pmod 5$

$98 - (x+4) = 5n$, for some integer n

$98 - x - 4 = 5n$

$94 - x = 5n$

$94 - x$ is a multiple of 5.

$\therefore$ The least positive value of x must be 4

$\because$ $94 - 4 = 90$ is the nearest multiple of 5 less than 94.

Example 2.14

Solve $8x \equiv 1 \pmod{11}$

Sol

$8x \equiv 1 \pmod{11}$ can be written as

$8x - 1 = 11k$, for some integer k.

$$8x = 11k + 1$$

$$x = \frac{11k+1}{8}$$

when we put $k = 5, 13, 21, 29, \ldots$ then $11k+1$ is divisible by 8.

$$x = \frac{11 \times 5 + 1}{8} = \frac{55+1}{8} = \frac{56}{8} = 7$$

$$x = \frac{11 \times 13 + 1}{8} = \frac{143+1}{8} = \frac{144}{8} = \cancel{64}\ 18$$

$\therefore$ the solutions are $7, 18, 29, 40, \ldots$

Example 2.15

compute x, such that $10^4 \equiv x \pmod{19}$

Sol

$10^2 = 100 \equiv 5 \pmod{19}$

$(10)^4 = (10^2)^2 \equiv 5^2 \pmod{19}$

$10^4 \equiv 25 \pmod{19}$

$10^4 \equiv 6 \pmod{19}$ $\quad [\because 25 \equiv 6 \pmod{19}]$

$\therefore x = 6$.

Example 2.16

Find the number of integers solutions of $3x \equiv 1 \pmod{15}$

Sol

$3x \equiv 1 \pmod{15}$ can be written as

$3x - 1 = 15k$, for some integer k.

$$3x = 15k + 1$$

$$x = \frac{15k+1}{3}$$

$$x = \frac{15k}{3} + \frac{1}{3}$$

$$x = 5k + \frac{1}{3}$$

$\therefore$ $5k$ is an integer ; $5k + \frac{1}{3}$ cannot be an integer

So, there is no integer solution

Example 2.17.

A man starts his journey from chennai to Delhi by train. He starts at 22.30 hours on wednesday. If it takes 32 hours of travelling time and assuming that the train is not late, when will he reach Delhi ?

Sol

Starting time 22.30. Travelling time 32 hours. Here we use modulo 24.

The reaching time is

$$22.30 + 32 \pmod{24} \equiv 54.30 \pmod{24}$$

$$\equiv 6.30 \pmod{24}$$

[$\because 32 = (1 \times 24) + 8$

Thursday Friday]

$\therefore$ He will reach Delhi on Friday at 6.30 hours.

Example 2.18

Kala and vani are friends. Kala says, "Today is my birthday" and she ask vani, "when will you celebrate your birthday ?" vani replies, "Today is Monday and I celebrated my birthday 15 days ago". Find the day when vani celebrated her birthday.

Sol

Let us associate the numbers 0, 1, 2, 3, 4, 5, 6 to represent the weekdays from sunday to saturday.

Vani says today is monday. So the number for monday is 1. Since vani's birthday was 75 days ago. we have to subtract 75 from 1 and take the modulo 7, since a week contain 7 days.

$$-74 \pmod 7 \equiv -4 \pmod 7$$
$$\equiv 7-4 \pmod 7$$
$$\equiv 3 \pmod 7$$

Thus, $1-75 \equiv 3 \pmod 7$ [$\because -74-3 = -77$ is divisible by 7)

The day for the number 3 is wednesday.

$\therefore$ Vani's birthday must be on Wednesday.

Example 2.19

Find the next three terms of sequences

(i) $\frac{1}{2}, \frac{1}{6}, \frac{1}{10}, \frac{1}{14}, \ldots$ (ii) $5, 2, -1, 4, \ldots$ (iii) $1, 0.1, 0.01, \ldots$

Sol

(i) $\frac{1}{2}, \frac{1}{6}, \frac{1}{10}, \frac{1}{14}, \ldots$

$+4 \quad +4 \quad +4$

In the above sequence the numerator are same and the denominator is increased by 4.

So, the next three terms are $a_5 = \frac{1}{14+4} = \frac{1}{18}$

$$a_6 = \frac{1}{18+4} = \frac{1}{22}$$

$$a_7 = \frac{1}{22+4} = \frac{1}{26}.$$

(ii) $5, 2, -1, -4$

$-3 \quad -3 \quad -3$

Here Each terms is decreased by 3. So the next three term are $-7, -10, -13, \ldots$

(iii) $1, \quad 0.1, \quad 0.01$

$\sqrt{10} \qquad \sqrt{10}$

Here each term is divided by 10. Hence, the next three terms are

$$a_4 = \frac{0.01}{10} = 0.001$$

$$a_5 = \frac{0.001}{10} = 0.0001$$

$$a_6 = \frac{0.0001}{10} = 0.00001$$

Example 2.20

Find the general terms for the following sequence.

(i) $3, 6, 9, \ldots$ (ii) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots$ (iii) $5, -25, 125, \ldots$

Sol (i) $3, 6, 9, \ldots$

Here the terms are multiple of 3. So the general term is

$$a_n = 3n.$$

(ii) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots$

$$a_1 = \frac{1}{2}; \quad a_2 = \frac{2}{3}; \quad a_3 = \frac{3}{4}$$

We see that the numerator of nth term is n, and the denominator is one more than the numerator. Hence

$$a_n = \frac{n}{n+1}, \; n \in N.$$

(iii) $5, -25, 125, \ldots$

The terms of the sequence have + and − sign alternatively and also they are in power of 5. So the general term

$$a_n = (-1)^{n+1} 5^n, \; n \in N.$$

Example 2.21

The general term of the sequence is defined as

$$a_n = \begin{cases} n(n+3) & ; n \in N \text{ is odd} \\ n^2+1 & ; n \in N \text{ is even.} \end{cases}$$

Find the eleventh and Eighteenth terms

Sol

To find a_{11}, since 11 is odd

we put $n=11$ in $a_n = n(n+3)$

The eleventh term,

$$a_{11} = 11(11+3) = 11(14) = 154$$

to find a_{18}, Since 18 is even

$n=18$ in $a_n = n^2+1$

The Eighteenth term,

$$a_{18} = (18)^2+1 = 324+1 = 325.$$

Example 2.22.

Find the first five terms of the following sequence.

$$a_1 = 1,\ a_2 = 1,\ a_n = \frac{a_{n-1}}{a_{n-2}+3} ;\ n \geq 3,\ n \in N.$$

Sol

The first two terms of this sequence are given by $a_1=1$, $a_2=1$, the third term a_3 depend on the first and second terms.

$$a_3 = \frac{a_{3-1}}{a_{3-2}+3} = \frac{a_2}{a_1+3} = \frac{1}{1+3} = \frac{1}{4}$$

Similarly, the fourth term a_4 depends upon a_2 and a_3

$$a_4 = \frac{a_{4-1}}{a_{4-2}+3} = \frac{a_3}{a_2+3} = \frac{\frac{1}{4}}{1+3} = \frac{\frac{1}{4}}{4} = \frac{1}{4} \times \frac{1}{4}$$

$$a_4 = \frac{1}{16}$$

The fifth term a_5 can be calculated

$$a_5 = \frac{a_{5-1}}{a_{5-2}+3} = \frac{a_4}{a_3+3} = \frac{\frac{1}{16}}{\frac{1}{4}+3}$$

$$a_5 = \frac{\frac{1}{16}}{\frac{1+12}{4}}$$

$$a_5 = \frac{\frac{1}{16}}{\frac{13}{4}}$$

$$a_5 = \frac{1}{16} \times \frac{4}{13} = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$$

$\therefore$ The first five terms of the sequence are

$1, 1, \frac{1}{4}, \frac{1}{16}$ and $\frac{1}{52}$

Example 2.23

Check whether the following sequence are in A.P or not ?

(i) $x+2, 2x+3, 3x+4, \ldots$ (ii) $2, 4, 8, 16, \ldots$

(iii) $3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}, 9\sqrt{2}, \ldots$

Sol

To check the given sequence as in A.P. It is enough to check if the differences between the consecutive terms are equal or not.

(i) $t_2 - t_1 = (2x+3) - (x+2) = 2x+3-x-2 = x+1$

$t_3 - t_2 = (3x+4) - (2x+3) = 3x+4-2x-3 = x+1$

$t_2 - t_1 = t_3 - t_2$

$\therefore$ the difference between consecutive terms are equal.

Hence the sequence $x+2, 2x+3, 3x+4, \ldots$ is in A.P.

(ii) $t_2 - t_1 = 4-2 = 2$

$t_3 - t_2 = 8-4 = 4$

$t_2 - t_1 \neq t_3 - t_2$

$\therefore$ The difference between consecutive terms are not equal. Hence the terms of the sequences

2, 4, 8, 16, ... are not in A.P.

(iii) $t_2 - t_1 = 5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$

$t_3 - t_2 = 7\sqrt{2} - 5\sqrt{2} = 2\sqrt{2}$

$t_4 - t_3 = 9\sqrt{2} - 7\sqrt{2} = 2\sqrt{2}$

$\therefore$ The difference between consecutive terms are equal. Hence the terms of the sequence $3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}, 9\sqrt{2}, \ldots$ are in A.P.

Example 2.24

Write an A.P whose first term is 20 and common difference is 8.

Sol

First term $= a = 20$;

common difference $= d = 8$

Arithmetic progression is $a, a+d, a+2d, \ldots$

$20, 20+8, 20+2(8), \ldots$

$20, 28, 20+16, \ldots$

So, the required AP is 20, 28, 36, ...

Example 2.25

Find the 15th, 24th and nth term (general term) of an A.P given by 3, 15, 27, 29, ...

Sol

First term $= a = 3$, common difference $= d = 15 - 3 = 12$

nth term (general term) of an A.P with first term a and common difference d is given by

$n = 15$, $t_n = a + (n-1)d$

$t_{15} = 3 + (15-1)12$

$t_{15} = 3 + (14)12$

$$t_{15} = 3 + 168$$

$$t_{15} = 171$$

$n = 24$,

$$t_{24} = 3 + (24-1)(12)$$

$$t_{24} = 3 + (23)(12)$$

$$t_{24} = 3 + 276$$

$$t_{24} = 279$$

The nth (general term) term is given by

$$t_n = a + (n-1)d$$

$$t_n = 3 + (n-1)12$$

$$t_n = 3 + 12n - 12$$

$$t_n = 12n - 9$$

Example 2.26

Find the number of terms in the A.P 3, 6, 9, 12, ..., 111

Sol

First term $= a = 3$; common difference $= d = 6 - 3 = 3$;

last term $l = 111$

$$n = \left(\frac{l-a}{d}\right) + 1$$

$$n = \left(\frac{111-3}{3}\right) + 1$$

$$= \left(\frac{108}{3}\right) + 1$$

$$= 36 + 1$$

$$n = 37$$

Thus the A.P contain 37 terms.

Example 2.27

Determine the general terms of an A.P whose 7th term is -1 and 16th term is 17.

Sol

Let the A.P be $t_1, t_2, t_3, t_4, \ldots$

Given: $t_7 = -1$ and $t_{16} = 17$

$$t_7 = a + (n-1)d$$

$$-1 = a + (7-1)d$$

$$-1 = a + 6d \longrightarrow (1)$$

$$t_{16} = a + (n-1)d$$

$$17 = a + (16-1)d$$

$$17 = a + 15d \longrightarrow (2)$$

subtract (2) - (1)

$$17 = a + 15d$$

$$-1 = a + 6d$$

$$(+) \quad (-) \quad (-)$$

$$18 = 9d$$

$$d = 18/9 = 2$$

Put $d = 2$ in (1)

$$-1 = a + 6(2)$$

$$-1 = a + 12$$

$$a = -1 - 12$$

$$\boxed{a = -13}$$

General term

$$t_n = a + (n-1)d$$

$$t_n = -13 + (n-1)2$$

$$t_n = -13 + 2n - 2$$

$$t_n = 2n - 15$$

Example 2.28

If l^{th}, m^{th}, n^{th} terms of an A.P are x, y, z respectively then show that

(i) $x(m-n) + y(n-l) + z(l-m) = 0$

(ii) $(x-y)n + (y-z)l + (z-x)m = 0$

Sol (1) Let a be the first term and d be the common difference. It is given that

35

$$t_l = x, \quad t_m = y, \quad t_n = z$$

General term

$$\boxed{t_n = a + (n-1)d}$$

$$t_l = a + (n-1)d$$

$$x = a + (l-1)d \rightarrow (1)$$

$$t_m = a + (n-1)d$$

$$y = a + (m-1)d \longrightarrow (2)$$

$$t_n = a + (n-1)d$$

$$z = a + (n-1)d \longrightarrow (3)$$

we have, $x(m-n) + y(n-l) + z(l-m)$

put x, y, z value using equation (1), (2), (3)

$$= (a+(l-1)d)(m-n) + (a+(m-1)d)\cancel{(m-n)}(n-l) + (a+(n-1)d)(l-m)$$

$$= a(m-n) + d(l-1)(m-n) + a(n-l) + d(m-1)(n-l) + a(l-m) + d(n-1)(l-m)$$

$$= a[(m-n)+(n-l)+(l-m)] + d[(l-1)(m-n)+(m-1)(n-l) + (n-1)(l-m)]$$

$$= a[m-n+n-l+l-m] + d[(lm-ln-m-n)+(mn-ml-n+l)+(nl-mn-l+m)]$$

$$= a[0] + d[lm-ln-m-n+mn-ml-n+l+nl-mn-l+m]$$

$$= a[0] + d[0] = 0+0 = 0$$

$$\therefore x(m-n) + y(n-l) + z(l-m) = 0$$

(ii) $(x-y)n + (y-z)l + (z-x)m = 0$

using equation ①, ② and ③

subtract ① − ②

$$x = a + (l-1)d$$

$$y = a + (m-1)d$$

$$x - y = (a-a) + ((l-1)d - (m-1)d)$$

$$x - y = 0 + ld - d - md + d$$

$$x - y = ld - md$$

$$x - y = (l-m)d \longrightarrow ④$$

Subtract ② − ③

$$y = a + (m-1)d$$

$$z = a + (n-1)d$$

$$y - z = (a-a) + ((m-1)d - (n-1)d)$$

$$y - z = 0 + (md - d - nd + d)$$

$$y - z = md - nd$$

$$y - z = (m-n)d \longrightarrow ⑤$$

Subtract ③ − ①

$$z = a + (n-1)d$$

$$x = a + (l-1)d$$

$$z - x = (a-a) + [(n-1)d - (l-1)d]$$

$$z - x = 0 + [nd - d - ld + d]$$

$$z - x = nd - ld$$

$$z - x = (n-l)d \longrightarrow ⑥$$

using sub ④, ⑤, ⑥ equation

$$(x-y)n + (y-z)l + (z-x)m$$

$$= [(l-m)d]n + [(m-n)d]l + [(n-l)d]m$$

$$= [(l-m)n + (m-n)l + (n-l)m]d$$

$$= [ln - mn + ml - nl + mn - ml]d$$

$$= (0)d = 0$$

Example 2.29

In an A.P sum of four consecutive terms is 28 and their sum of their squares is 276. Find the four numbers.

Sol

Let us take the four terms in the form $(a-3d)$, $(a-d)$, $(a+d)$ and $(a+3d)$.

Since, sum of the four terms is 28

$$a - 3d + a - d + a + d + a + 3d = 28$$

$$4a = 28$$

$$a = 28/4$$

$$\boxed{a = 7}$$

Similarly, since sum of their square is 276.

$$(a-3d)^2 + (a-d)^2 + (a+d)^2 + (a+3d)^2 = 276$$

$$a^2 - 6ad + 9d^2 + a^2 - 2ad + d^2 + a^2 + 2ad + d^2 + a^2 + 6ad + 9d^2 = 276$$

$$4a^2 + 20d^2 = 276$$

$$4(7)^2 + 20d^2 = 276$$

$$4(49) + 20d^2 = 276$$

$$196 + 20d^2 = 276$$

$$20d^2 = 276 - 196$$

$$20d^2 = 80$$

$$d^2 = \frac{80}{20} = 4$$

$$d = \sqrt{4}$$

$$d = \pm 2$$

If $d = 2$, then the four numbers are $7 - 3(2), 7 - 2, 7 + 2, 7 + (3)(2)$

$\therefore$ The four numbers are 1, 5, 9 and 13

If $a=7$, $d=-2$, then the four numbers are

$7-3(-2)$, $7-(-2)$, $7-2$, $7+3(-2)$

$\therefore$ The four numbers are 13, 9, 5, 1

$\therefore$ the four consecutive terms of the A.P are 1, 5, 9, 13.

Example 2.30

A mother divides ₹ 207 into three parts such that the amount are in A.P and gives it to her three children. the product of the two least amounts that the children had ₹ 4623. Find the amount received by each child.

Sol

Let the amount received by the three children be in the form of A.P is given by

$a-d, a, a+d.$

Since, sum of the amount is ₹ 207.

$$(a-d)+a+(a+d) = 207$$

$$a-d+a+a+d = 207$$

$$3a = 207$$

$$a = \frac{207}{3}$$

$$\boxed{a = 69}$$

It is given that product of the two least amount is 4623.

$$(a-d)a = 4623$$

$$(69-d)69 = 4623$$

$$4761 - 69d = 4627$$

$$-69d = 4627 - 4761$$

$$-69d = -134$$

$$d = \frac{-134}{-69} = 1.94$$

$$\boxed{d = 2}$$

$\therefore$ Amount given by the mother to her three children are

₹(69 - 2), ₹ 69, ₹(69 + 2)

$\therefore$ ₹ 67, ₹ 69, ₹ 71

Example 2.31

Find the sum of first 15 terms of the A.P 8, $7\frac{1}{4}$, $6\frac{1}{2}$, $5\frac{3}{4}$, ...

Sol Here the first term $a = 8$, common difference

$$d = 7\frac{1}{4} - 8$$

$$d = \frac{29}{4} - 8$$

$$d = \frac{29 - 32}{4} = -\frac{3}{4}$$

Sum of first n terms of an A.P

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{15} = \frac{15}{2}\left[2(8) + (15-1)\left(-\frac{3}{4}\right)\right]$$

$$= \frac{15}{2}\left[16 + 14\left(-\frac{3}{4}\right)\right]$$

$$= \frac{15}{2}\left[16 - \frac{42}{4}\right]$$

$$= \frac{15}{2}\left[16 - \frac{21}{2}\right]$$

$$= \frac{15}{2}\left[\frac{32-21}{2}\right]$$

$$= \frac{15}{2}\left[\frac{11}{2}\right] = \frac{165}{4}$$

Example 2.32

Find the sum of 0.40 + 0.43 + 0.46 + ... + 1

Sol

Here, the value of n is not given. But the last term is given. From this, we can find the value of n.

(40)

Given: $a = 0.40$, $l = 1$, $d = 0.43 - 0.40 = 0.03$

$$\therefore \quad n = \left[\frac{l-a}{d}\right] + 1$$

$$= \left[\frac{1-0.40}{0.03}\right] + 1$$

$$= \left[\frac{0.6}{0.03}\right] + 1$$

$$= \left[\frac{0.6}{0.03} \times \frac{100}{100}\right] + 1$$

$$= \left[\frac{60}{3}\right] + 1$$

$$= 20 + 1$$

$$n = 21$$

Sum of first n terms of an A.P.

$$S_n = \frac{n}{2}[a + l]$$

$$S_{21} = \frac{21}{2}[0.40 + 1]$$

$$= \frac{21}{2}[1.40]$$

$$= 10.5\,(1.40)$$

$$= \cancel{14}\ 14.7$$

So, the sum of 21 terms of the given series is 14.7.

Example 2.33

How many terms of the series $1 + 5 + 9 + \ldots$ must be taken so that their sum is 190?

Sol

to find the value of n, $S_n = 190$

First term $a = 1$, common difference $d = 5 - 1 = 4$

Sum of first n terms of an A.P

$$S_n = \frac{n}{2}[2a + (n-1)d] = 190$$

$$\frac{n}{2}[2a + (n-1)d] = 190$$

$$\frac{n}{2}[2(1) + (n-1)4] = 190$$

$$\frac{n}{2}[2 + 4n - 4] = 190$$

$$\frac{n}{2}[4n - 2] = 190$$

$$n[4n - 2] = 190 \times 2$$

$$n[4n - 2] = 380$$

$$4n^2 - 2n = 380$$

$$4n^2 - 2n - 380 = 0$$

$$2n^2 - n - 190 = 0$$

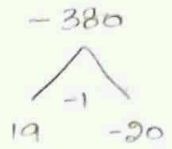
$$2n^2 + 19n - 20n - 190 = 0$$

$$n(2n + 19) - 10(2n + 19) = 0$$

$$(n - 10)(2n + 19) = 0$$

$n = 10$, $n = \frac{-19}{2}$ is impossible

$$\therefore n = 10$$



Example 2.34

The 13th term of an A.P is 3 and the sum of first 13 terms is 234. Find the common difference and the sum of first 21 terms.

Sol Given: 13th term = 3

$$t_n = a + (n-1)d$$

$$t_{13} = a + (13-1)d$$

$$3 = a + 12d \longrightarrow (1)$$

Sum of first 13 terms = 234

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{13} = \frac{13}{2}[2a + (13-1)d]$$

$$234 = \frac{13}{2}[2a + 12d]$$

$$468 = 13[2a + 12d]$$

$$\frac{468}{13} = 2a + 12d$$

$$36 = 2a + 12d \longrightarrow (2)$$

Solving (1) & (2)

$$3 = a + 12d$$

$$36 = 2a + 12d$$

$$\ominus \quad \ominus \quad \ominus$$

$$-33 = -a$$

$$\boxed{a = 33}$$

$a = 33$ in (1)

$$3 = 33 + 12d$$

$$3 - 33 = 12d$$

$$-30 = 12d$$

$$d = \frac{-30}{12} = \frac{-5}{2}$$

$\therefore$ common difference is $\frac{-5}{2}$

Sum of first 21 terms

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{21} = \frac{21}{2}\left[2(33) + (21-1)\left(\frac{-5}{2}\right)\right]$$

$$= \frac{21}{2}\left[66 + (20)\left(-\frac{5}{2}\right)\right]$$

$$= \frac{21}{2}[66 + (10)(-5)]$$

$$= \frac{21}{2}[66-50]$$

$$= \frac{21}{2}[16]$$

$$= 21(8)$$

$$S_{21} = 168$$

Example 2.35

In an A.P the sum of first n terms is $\frac{5n^2}{2} + \frac{3n}{2}$

Find the 17th term.

Sol The 17th term can be obtained by subtracting the sum of first 16 terms from the sum of first 17th terms.

$$S_n = \frac{5n^2}{2} + \frac{3n}{2}$$

$$S_{17} = \frac{5\times(17)^2}{2} + \frac{3\times 17}{2} = \frac{5\times 289}{2} + \frac{51}{2}$$

$$= \frac{1445}{2} + \frac{51}{2}$$

$$S_{17} = \frac{1496}{2} = 748$$

$$S_{16} = \frac{5\times(16)^2}{2} + \frac{3\times 16}{2} = \frac{5\times 256}{2} + \frac{48}{2}$$

$$= \frac{1280}{2} + \frac{48}{2}$$

$$S_{16} = \frac{1328}{2} = 664$$

$$t_{17} = S_{17} - S_{16} = 748 - 664 = 84$$

Example 2.36

Find the sum of all natural numbers between 300 and 600 which are divisible by 7.

Sol The natural numbers between 300 and 600 which are divisible by 7 are 301, 308, 315, ..., 595

The sum of all natural numbers between 300 and 600 is $301 + 308 + 315 + \cdots + 595$ (44)

The terms of the above series are in A.P

First term $a = 301$, common difference $d = 7$;
last term $l = 595$

$$n = \left[\frac{l-a}{d}\right] + 1$$

$$= \left[\frac{595 - 301}{7}\right] + 1$$

$$= \left[\frac{294}{7}\right] + 1$$

$$= 42 + 1$$

$$n = 43$$

$$S_n = \frac{n}{2}[a + l]$$

$$S_{43} = \frac{43}{2}[301 + 595]$$

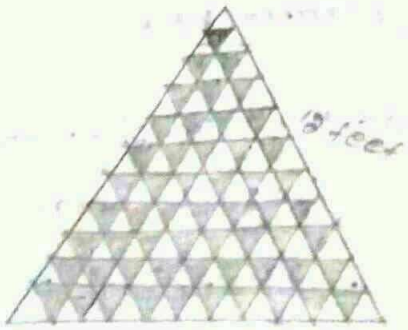
$$= \frac{43}{2}[896]$$

$$= 43(448)$$

$$S_{43} = 19264$$

Example 2.37

A mosaic is designed in the shape of an equilateral triangle, 12 ft on each side. Each tile in the mosaic is in the shape of an equilateral triangle of 12 inch side. the tile are alternate in colour as shown in figure. Find the number of tiles of each colour and total number of tiles in the mosaic.



Sol Since the mosaic is in the shape of an equilateral triangle of 12 feet and the tiles is in the shape of an equilateral triangle of 12 inch (1 feet), there will be 12 rows in the mosaic.

From the figure, it is clear that number of white tiles in each row are 1, 2, 3, 4, ..., 12, which clearly forms an Arithmetic progression.

Similarly the number of blue tiles in each row are 0, 1, 2, 3, ..., 11 which is also an Arithmetic progression.

No. of white tiles $= 1+2+3+\cdots+12 = \frac{12}{2}[1+12]$

$= \frac{12}{2}[13]$

$= 6[13]$

$= 78$

No. of blue tiles $= 0+1+2+\cdots+11 = \frac{12}{2}[0+11]$

$= \frac{12}{2}[11]$

$= 6[11]$

$= 66$

The total number of tiles in the mosaic $= 78+66 = 144$

Example 2.38

The house of a street are numbered from 1 to 49. Senthil's house is numbered such that the sum of number of the houses prior to senthil house is equal to the sum of numbers of the houses following senthil's house. Find senthil's house number?

Sol Let senthil's house number be x

Given:

$1+2+3+\cdots+(x-1) = (x+1)+(x+2)+\cdots+49$

$1+2+3+\cdots+(x-1) = [1+2+3+\cdots+49] - [1+2+3+\cdots+x]$

$\frac{x-1}{2}[1+(x-1)] = \frac{49}{2}[1+49] - \frac{x}{2}[1+x]$

$$\frac{x-1}{2}[1+x-1] = \frac{49}{2}[50] - \frac{x}{2}[1+x]$$

$$\frac{x(x-1)}{2} = \frac{49\times 50}{2} - \frac{x(1+x)}{2}$$

$$x(x-1) = 2450 - x(1+x)$$

$$x^2 - x = 2450 - x^2 - x$$

$$x^2 + x^2 - x + x = 2450$$

$$2x^2 = 2450$$

$$x^2 = \frac{2450}{2}$$

$$x^2 = 1225$$

$$x = 35$$

$\therefore$ Senthil's house number is 35.

Example 2.39

The sum of first n, 2n and 3n terms of an A.P are S_1, S_2 and S_3 respectively. prove that $S_3 = 3(S_2 - S_1)$

Sol:

If S_1, S_2, S_3 are sum of first n, 2n, 3n terms of an A.P respectively.

$$S_1 = \frac{n}{2}[2a + (n-1)d]$$

$$S_2 = \frac{2n}{2}[2a + (2n-1)d]$$

$$S_3 = \frac{3n}{2}[2a + (3n-1)d]$$

$$S_2 - S_1 = \frac{2n}{2}[2a + (2n-1)d] - \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[4a + 2(2n-1)d] - \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[4a - 2a + 2(2n-1)d - 1(n-1)d]$$

$$= \frac{n}{2}[2a + (4n - 2 - n + 1)d]$$

$$S_2 - S_1 = \frac{n}{2}[2a + (3n-1)d]$$

$$3(S_2 - S_1) = \frac{3n}{2}[2a + (3n-1)d]$$

$$3(S_2 - S_1) = S_3$$

Example 2.40

which of the following sequences form a Geometric progression?

(i) 7, 14, 21, 28, ... (ii) $\frac{1}{2}$, 1, 2, 4, ... (iii) 5, 25, 50, 75, ...

Sol

To check if a given sequence form an G.P we have to see if the ratio between successive terms are equal.

(i) 7, 14, 21, 28, ...

$$\frac{t_2}{t_1} = \frac{14}{7} = 2; \quad \frac{t_3}{t_2} = \frac{21}{14} = \frac{3}{2}; \quad \frac{t_4}{t_3} = \frac{28}{21} = \frac{4}{3}$$

Since the ratio between successive terms are not equal, the sequence 7, 14, 21, 28, ... is not a Geometric progression.

(ii) $\frac{1}{2}$, 1, 2, 4, ...

$$\frac{t_2}{t_1} = \frac{1}{\frac{1}{2}} = 2; \quad \frac{t_3}{t_2} = \frac{2}{1} = 2; \quad \frac{t_4}{t_3} = \frac{4}{2} = 2$$

The ratio between successive terms are equal.

$\therefore$ The sequence $\frac{1}{2}$, 1, 2, 4, ... is a Geometric progression with common ratio is 2.

(iii) 5, 25, 50, 75

$$\frac{t_2}{t_1} = \frac{25}{5} = 5; \quad \frac{t_3}{t_2} = \frac{50}{25} = 2; \quad \frac{t_4}{t_3} = \frac{75}{50} = \frac{3}{2}$$

Since the ratio between successive terms are not equal. the sequence 5, 25, 50, 75, ... is not a Geometric progression.

Example 2.41

Find the Geometric progression whose first term and common ratio are given by (i) $a=-7$, $r=6$ (ii) $a=256$, $r=0.5$

Sol

(i) The General form of Geometric progression is $a, ar, ar^2, \ldots$

$$a=-7, \quad ar=-7\times 6=-42, \quad ar^2=-7\times(6)^2=-7\times 36$$
$$=-252$$

$\therefore$ Geometric progression is $-7, -42, -252, \ldots$

(ii) The General form of Geometric progression is a, ar, ar^2

$$a=256, \quad ar=256\times 0.5=128, \quad ar^2=256\times(0.5)^2$$
$$=256\times 0.25$$
$$=64$$

$\therefore$ Geometric progression is $256, 128, 64, \ldots$

Example 2.42

Find the 8th term of the G.P $9, 3, 1, \ldots$

Sol

To find 8th term, using nth term formula

$$t_n = ar^{n-1}.$$

First term $a=9$, common difference $r=\frac{t_2}{t_1}=\frac{3}{9}=\frac{1}{3}$

$$t_8 = 9\left(\frac{1}{3}\right)^{8-1}$$
$$=9\left(\frac{1}{3}\right)^7$$
$$t_8 = 243$$

$\therefore$ 8th term of the G.P is 243

Example 2.43

In a Geometric progression, the 4th term is $\frac{8}{9}$, and the 7th term is $\frac{64}{243}$. Find the Geometric progression.

Sol

4th term, $t_4 = \frac{8}{9} \Rightarrow ar^3 = \frac{8}{9} \rightarrow (1)$

7th term, $t_7 = \frac{64}{243} \Rightarrow ar^6 = \frac{64}{243} \rightarrow (2)$

dividing (2) by (1)

$$\frac{ar^6}{ar^3} = \frac{\frac{64}{243}}{\frac{8}{9}}$$

$$r^3 = \frac{64}{243} \times \frac{9}{8}$$

$$r^3 = \frac{8}{27}$$

$$r^3 = \frac{2^3}{3^3}$$

$$r = \frac{2}{3}$$

Substitute $r = 2/3$ in (1)

$$a(2/3)^3 = 8/9$$

$$a\left(\frac{8}{27}\right) = \frac{8}{9}$$

$$a = \frac{8}{9} \times \frac{27}{8}$$

$$\boxed{a = 3}$$

$\therefore$ Geometric progression is $a, ar, ar^2, \ldots$

G.P is $3, 3(2/3), 3\left(\frac{2}{3}\right)^2$

G.P is $3, 2, \frac{4}{3}$

Example 2.44

The product of three consecutive terms of a Geometric progression is 343 and their sum is $\frac{91}{3}$. Find the three terms.

30) Since the product of 3 consecutive terms is given.

we can take them as $\frac{a}{r}, a, ar,$ (150)

Product of the terms $= 343$

$$\frac{a}{r} \times a \times ar = 343$$

$$a^3 = 343$$

$$a^3 = 7^3$$

$$\boxed{a = 7}$$

Sum of the terms $= 91/3$

$$\left[\frac{a}{r} + a + ar\right] = \frac{91}{3}$$

$$a\left[\frac{1}{r} + 1 + r\right] = \frac{91}{3}$$

$$7\left[\frac{1+r+r^2}{r}\right] = \frac{91}{3}$$

$$\left[\frac{1+r+r^2}{r}\right] = \frac{91}{3 \times 7}$$

$$3(1+r+r^2) = 13r$$

$$3+3r+3r^2 = 13r$$

$$3r^2 - 13r + 3r + 3 = 0$$

$$3r^2 - 10r + 3 = 0$$

$$3r^2 - r - 9r + 3 = 0$$

$$r(3r-1) - 3(3r-1) = 0$$

$$(r-3)(3r-1) = 0$$

$$r = 3 \; ; \; r = \frac{1}{3}$$

If $a = 7, r = 3$, then the three terms are $\frac{7}{3}, 7, 21$

If $a = 7, r = \frac{1}{3}$, then the three terms are $21, 7, \frac{7}{3}$.

Example 2.45

The present value of a machine is ₹40,000 and its value depreciates each year by 10%. Find the estimate value of the machine in the 6th year.

Sol

The value of the machine at present is ₹40,000 Since it is depreciated at the rate of 10% after one year the value of the machine 90% of the initial value.

The value of the machine at the end of the first year is $40,000 \times \frac{90}{100}$

After two year, the value of the machine is 90% of the value in the first year.

Value of the machine at the end of the 2nd year is $40,000 \times \left(\frac{90}{100}\right)^2$

Continuing this way, the value of the machine depreciates in the following way as

$$40,000 \times 40,000 \times \left(\frac{90}{100}\right), 40000 \times \left(\frac{90}{100}\right)^2, \ldots$$

This sequence is in the form of G.P with first term 40,000 and common ratio $\frac{90}{100}$.

Finding the value of the machine at the end of 5th year (i.e. in 6th year). to find 6th term of G.P

$n = 6,\ a = 40,000,\ r = \frac{90}{100}$

$t_n = ar^{n-1}$

$t_6 = 40,000 \times \left(\frac{90}{100}\right)^{6-1}$

$= 40,000 \times \left(\frac{90}{100}\right)^5$

$t_6 = 40,000 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}$

$t_6 = 23619.6$

$\therefore$ the value of the machine in 6th year = ₹ 23619.6

Example 2.46

Find the sum of 8 terms of the G.P. 1, -3, 9, -27, ...

Sol

The first term $a = 1$, common ratio $r = \frac{t_2}{t_1} = \frac{-3}{1} = -3 < 1$

$n = 8$.

Sum to n terms of a G.P is

$$S_n = \frac{a(r^n - 1)}{r - 1} \text{ if } r \neq 1$$

$$S_8 = \frac{1((-3)^8 - 1)}{-3-1} = \frac{1(6561-1)}{-4} = \frac{6560}{-4}$$

$$S_8 = -1640$$

Example 2.47

Find the first terms of the G.P in which $S_6 = 4095$ and $r = 4$.

Sol

Common ratio $= 4 > 1$, sum of first 6 terms $S_6 = 4095$

$$S_6 = \frac{a(r^n - 1)}{r - 1} = 4095$$

$$r = 4,\ 4095 = \frac{a(4^6 - 1)}{4 - 1} = \frac{a(4096 - 1)}{3}$$

$$4095 = a \times \frac{4095}{3}$$

$$a = 4095 \times \frac{3}{4095}$$

$$\boxed{a = 3}$$

Example 2.48

How many terms of the series $1+4+16+\cdots$ make the sum 1365 ?

Sol

Let n be the number of terms to be added to get the sum 1365.

$$a = 1, \quad r = \frac{4}{1} = 4 > 1$$

$$S_n = 1365 \Rightarrow \frac{a(r^n - 1)}{r-1} = 1365$$

$$\frac{1(4^n - 1)}{4-1} = 1365$$

$$4^n - 1 = 1365 \times 3$$

$$4^n - 1 = 4095$$

$$4^n = 4095 + 1$$

$$4^n = 4096$$

$$4^n = 4^6$$

$$\boxed{n = 6}$$

Example 2.49

Find the sum $3 + 1 + \frac{1}{3} + \cdots \infty$

Sol

$$a = 3, \quad r = \frac{t_2}{t_1} = \frac{1}{3}$$

sum of infinite term

$$S_\infty = \frac{a}{1-r} = \frac{3}{1-\frac{1}{3}} = \frac{3}{\frac{3-1}{3}} = \frac{3}{\frac{2}{3}} = 3 \times \frac{3}{2}$$

$$S_\infty = \frac{9}{2}$$

Example 2.50

Find the rational form of the number 0.6666...

Sol

We can express the number 0.6666

$$0.6666... = 0.6 + 0.06 + 0.006 + 0.0006 + \cdots$$

number 0.6, 0.06, 0.006, ... form a G.P

first term $a = 0.6$, common ratio $r = \frac{0.06}{0.6} = 0.1$

Also $-1 < r = 0.1 < 1$

using the infinite G.P formula.

$$0.6666\ldots = 0.6 + 0.06 + 0.006 + \cdots = \frac{0.6}{1-0.1} = \frac{0.6}{0.9} = \frac{2}{3}$$

$\therefore$ The rational number equivalent of 0.6666... is $\frac{2}{3}$

Example 2.51

Find the sum to n terms of the series $5 + 55 + 555 + \ldots$,

Sol

The series is neither Arithmetic nor Geometric series. So it can be split into two series and then find the sum.

$$5 + 55 + 555 + \cdots + n \text{ terms} = 5[1 + 11 + 111 + \cdots + n \text{ terms}]$$

$$= \frac{5}{9}[9 + 99 + 999 + \cdots + n \text{ terms}]$$

$$= \frac{5}{9}[(10-1) + (100-1) + (1000-1) + \cdots + n \text{ terms}]$$

$$= \frac{5}{9}[(10 + 100 + 1000 + \cdots + n \text{ terms}) - n]$$

$$= \frac{5}{9}\left[\frac{10(10^n - 1)}{10-1} - n\right] = \frac{50}{81}(10^n - 1) - \frac{5n}{9}$$

Example 2.52

Find the least positive integer n such that $1 + 6 + 6^2 + \cdots + 6^n > 5000$

Sol

to find the number of terms for which the sum must be greater than 5000

to find least value of n, $S_n > 5000$

$$S_n = \frac{a(r^n-1)}{r-1} = \frac{1(6^n-1)}{6-1} = \frac{6^n-1}{5}$$

$$S_n > 5000 = \frac{6^n-1}{5} > 5000$$

$$6^n - 1 > 25000 \Rightarrow 6^n > 25001$$

$\because$ $6^5 = 7776$ and $6^6 = 46656$

The least positive value of n is 6 such that

$1 + 6 + 6^2 + \cdots + 6^n > 5000$

Example 2.53

A person saved money every year, half as much as he could in the previous year. If he had totally saved ₹ 7875 in 6 years then how much did he save in the first year?

Sol

total amount saved in 6 years is $S_6 = 7875$

Since he saved half a much money as every year he saved in the previous year.

$$r = \frac{1}{2} < 1$$

$$\frac{a(1-r^n)}{1-r} = \frac{a\left(1-\left(\frac{1}{2}\right)^6\right)}{1-\frac{1}{2}} = 7875$$

$$\frac{a\left(1-\frac{1}{64}\right)}{\frac{1}{2}} = 7875$$

$$\frac{a\left(\frac{64-1}{64}\right)}{1/2} = 7875$$

$$a\left(\frac{63}{64}\right) \times \left(\frac{2}{1}\right) = 7875$$

$$a \times \frac{63}{32} = 7875$$

$$a = \frac{7875 \times 32}{63}$$

$$a = 4000$$

The amount saved in the first year is ₹ 4000

Example 2.54

Find the value of (i) $1+2+3+\cdots+50$ (ii) $16+17+18+\cdots+75$

Sol (i) $1+2+3+\cdots+50$

$$1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

$$1+2+3+\cdots+50 = \frac{50(50+1)}{2} = \frac{50(51)}{2} = 25(51)$$

$$= 1275$$

(ii) $16+17+18+\cdots+75$

$$= (1+2+3+\cdots+75) - (1+2+3+\cdots+15)$$

$$= \frac{75(75+1)}{2} - \frac{15(15+1)}{2}$$

$$= \frac{75(76)}{2} - \frac{15(16)}{2}$$

$$= 75(38) - 15(8)$$

$$= 2850 - 120 = 2730$$

$$= 2730$$

Example 2.55

Find the sum of (i) $1+3+5+\cdots$ to 40 terms

(ii) $2+4+6+\cdots+80$ (iii) $1+3+5+\cdots+55$

Sol

(i) $1+3+5+\cdots$ 40 terms $= 40^2 = 1600$

(ii) $2+4+6+\cdots+80 = 2(1+2+3+\cdots+40)$

$$= 2\left(\frac{40\times(40+1)}{2}\right) = 40\times 41 = 1640$$

(iii) $1+3+5+\cdots+55$

Here the number of terms is not given. To find the number of terms,

$$n = \frac{(l-a)}{d} + 1$$

$$n = \frac{55-1}{2} + 1 = \frac{54}{2} + 1 = 27+1 = 28$$

$$\therefore 1+3+5+\cdots+55 = (28)^2$$

$$= 784$$

Example 2.56

Find the sum of (i) $1^2+2^2+\cdots+19^2$

(ii) $5^2+10^2+15^2+\cdots+105^2$ (iii) $15^2+16^2+17^2+\cdots+28^2$

Sol

(i) $1^2+2^2+\cdots+19^2 = \frac{19\times(19+1)(2\times19+1)}{6}$

$$= \frac{19\times20\times39}{6} = 2470$$

(ii) $5^2+10^2+15^2+\cdots+105^2 = 5^2(1^2+2^2+\cdots+21^2)$

$$= 25\times\frac{21\times(21+1)(2\times21+1)}{6}$$

$$= \frac{25\times21\times22\times43}{6}$$

$$= 82775$$

(iii) $15^2+16^2+17^2+\cdots+28^2 = (1^2+2^2+3^2+\cdots+28^2) - (1^2+2^2+3^2+\cdots+14^2)$

$$= \frac{28\times(28+1)(2\times28+1)}{6} - \frac{14\times(14+1)(2\times14+1)}{6}$$

$$= \frac{28 \times 29 \times 57}{6} - \frac{14 \times 15 \times 29}{6}$$

$$= 7714 - 1015$$

$$= 6699$$

Example 2.57

Find the sum of (i) $1^3 + 2^3 + 3^3 + \cdots + 16^3$ (ii) $9^3 + 10^3 + \cdots + 21^3$

Sol (i) $1^3 + 2^3 + 3^3 + \cdots + 16^3 = \left[\frac{16 \times (16+1)}{2}\right]^2$

$$= \left[\frac{16 \times 17}{2}\right]^2 = [8 \times 17]^2$$

$$= (136)^2$$

$$= 18496$$

(ii) $9^3 + 10^3 + \cdots + 21^3 = (1^3 + 2^3 + 3^3 + \cdots + 21^3) - (1^3 + 2^3 + 3^3 + \cdots + 8^3)$

$$= \left[\frac{21 \times (21+1)}{2}\right]^2 - \left[\frac{8 \times (8+1)}{2}\right]^2$$

$$= \left[\frac{21 \times 22}{2}\right]^2 - \left[\frac{8 \times 9}{2}\right]^2$$

$$= [21 \times 11]^2 - [4 \times 9]^2$$

$$= [231]^2 - (36)^2$$

$$= 52065$$

Example 2.58

If $1 + 2 + 3 + \cdots + n = 666$ then find n.

Sol Since, $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$, $\frac{n(n+1)}{2} = 666$

$$\frac{n^2 + n}{2} = 666$$

$$n^2 + n = 1332$$

$$n^2 + n - 1332 = 0$$

-1332

$37 \quad 1 \quad -36$

$n^2 + 37n - 36n - 1332 = 0$

$n(n+37) - 36(n+37) = 0$

$(n+37)(n-36) = 0$

$n = -37, \; n = 36$

$n \neq -37$ ($\because$ n is a natural number)

Hence $n = 36$.

Xth HAND WRITTEN MATERIALS

CHAPTER – 3 (Examples)

ALGEBRA

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CHAPTER - 3

ALGEBRA

Example 3.1

The father's age is six times his son's age. Six years hence the age of the father will be four times his son's age. Find the present age (in years) of the son and father.

Sol

Let the present age of father be x years

Let the present age of son be y years

Given: $x = 6y \rightarrow (1)$

$x+6 = 4(y+6) \rightarrow (2)$

Substituting (1) in (2)

$6y+6 = 4(y+6)$

$6y+6 = 4y+24$

$6y-4y = 24-6$

$2y = 18$

$y = 18/2$

$\boxed{y=9} \rightarrow (3)$

Substitute (3) in (1)

$x = 6(9)$

$\boxed{x = 54}$

$\therefore$ Son's age = 9 years ; Father's Age = 54 years.

Example 3.2

Solve $2x-3y=6$; $x+y=1$

Sol

$2x-3y=6 \rightarrow (1)$

$x+y=1 \rightarrow (2)$

$(2)\times 2 \quad 2x+2y=2 \rightarrow (3)$

$$\begin{array}{r} 2x-3y=6 \\ 2x+2y=2 \\ \hline -5y=4 \end{array}$$

$$y = \frac{-4}{5}$$

Substitue $y = -\frac{4}{5}$ in ②

$$x - \frac{4}{5} = 1$$

$$\frac{5x-4}{5} = 1$$

$$5x = 5+4$$

$$5x = 9$$

$$x = 9/5$$

$$\therefore \quad x = \frac{9}{5} \; ; \; y = \frac{-4}{5}$$

Example 3.3

Solve the following system of linear equations in three variables.

$$3x - 2y + z = 2, \quad 2x + 3y - z = 5, \quad x + y + z = 6.$$

Sol

$$3x - 2y + z = 2 \longrightarrow ①$$

$$2x + 3y - z = 5 \longrightarrow ②$$

$$x + y + z = 6 \longrightarrow ③$$

Adding ① & ②

$$\begin{array}{rl} & 3x - 2y + z = 2 \\ (+) & \underline{2x + 3y - z = 5} \\ & \underline{5x + y \quad = 7} \longrightarrow ④ \end{array}$$

Adding ② & ③

$$\begin{array}{rl} & 2x + 3y - z = 5 \\ (+) & \underline{x + y + z = 6} \\ & \underline{3x + 4y \quad = 11} \longrightarrow ⑤ \end{array}$$

Equation ④ × 4

$$20x + 4y = 28 \longrightarrow ⑥$$

Subtract ⑥ & ⑤

$$20x + 4y = 28$$
$$3x + 4y = 11$$
$$\ominus \qquad \ominus \qquad \ominus$$
$$17x = 17$$
$$x = 17/17$$
$$\boxed{x = 1}$$

Substitute $x = 1$ in (4)

$$3(1) + 4y = 11$$
$$4y = 11 - 3$$
$$4y = 8$$
$$y = 8/4$$
$$\boxed{y = 2}$$

Substitute $x = 1, y = 2$ in (3)

$$1 + 2 + z = 6$$
$$3 + z = 6$$
$$z = 6 - 3$$
$$\boxed{z = 3}$$

$\therefore$ $x = 1, y = 2 ; z = 3$

Example 3.4

In a interschool atheletic meet, with total of 24 individual prices, securing a total of 56 points, a first place secure 5 points, a second place secure 3 points, and the third places secures 1 point. Having as many third place finishers as first and second place finishers find how many athletes finished in each place.

Sol Let the numbers of I, II, III place finishers be x, y and z respectively.

total number of events = 24 :

total number of points = 56 :

Hence, the linear equation in three variables are

$$x+y+z = 24 \longrightarrow (1)$$

$$5x+3y+z = 56 \longrightarrow (2)$$

$$x+y = z \longrightarrow (3)$$

substituting (3) in (1)

$$z+z = 24$$

$$2z = 24$$

$$z = 24/2$$

$$\boxed{z = 12}$$

$\therefore$ In (3) $x+y = 12$

(3) $\times 3 \Rightarrow$ $3x+3y = 36 \longrightarrow (4)$

substitute $z=12$ in (2)

$$5x+3y+12 = 56$$

$$5x+3y = 56-12$$

$$5x+3y = 44$$

solve (2) & (4)

$$5x+3y = 44$$

$$3x+3y = 36$$

$$\ominus \quad \ominus \quad \ominus$$

$$2x = 8$$

$$x = 8/2$$

$$\boxed{x = 4}$$

substitute $x=4$, $z=12$ in (3)

$$4+y = 12$$

$$y = 12-4$$

$$\boxed{y = 8}$$

$\therefore$ $x=4$, $y=8$, $z=12$

Number of first place finishers is 4

Number of second place finishers is 8

Number of third place finishers is 12.

Example 3.5

Solve $x+2y-z=5$; $x-y+z=-2$; $-5x-4y+z=-11$

Sol

$$x+2y-z=5 \longrightarrow (1)$$
$$x-y+z=-2 \longrightarrow (2)$$
$$-5x-4y+z=-11 \longrightarrow (3)$$

Adding (1) & (2)

$$\begin{aligned} x+2y-z &= 5 \\ x-y+z &= -2 \\ \hline 2x+y &= 3 \longrightarrow (4) \end{aligned}$$

subtract (2) & (3)

$$\begin{aligned} x-y+z &= -2 \\ -5x-4y+z &= -11 \\ (+)\quad (+)\quad (-)\quad (+) \\ \hline 6x+3y &= 9 \end{aligned}$$

divide by 3 $\quad 2x+y=3 \longrightarrow (5)$

subtracting (4) & (5)

$$\begin{aligned} 2x+y &= 3 \\ 2x+y &= 3 \\ (-)\quad (-)\quad (-) \\ \hline 0 &= 0 \end{aligned}$$

Here we arrive at an identity $0=0$

Hence the system has an infinite number of solution.

Example 3.6

Solve $3x+y-3z=1$; $-2x-y+2z=1$; $-x-y+z=2$.

Sol

$$3x+y-3z=1 \longrightarrow (1)$$
$$-2x-y+2z=1 \longrightarrow (2)$$

$$-x - y + z = 2 \longrightarrow (3)$$

Adding (1) & (2)

$$\begin{array}{r} 3x + y - 3z = 1 \\ -2x - y + 2z = 1 \\ \hline x \quad - z = 2 \\ \hline \end{array} \longrightarrow (4)$$

Adding (1) & (3)

$$\begin{array}{r} 3x + y - 3z = 1 \\ -x - y + z = 2 \\ \hline 2x \quad - 2z = 3 \\ \hline \end{array} \longrightarrow (5)$$

Eq (4) × 2

$$2x - 2z = 4 \longrightarrow (4)$$

Solve (5) & (4)

$$\begin{array}{r} 2x - 2z = 3 \\ 2x - 2z = 4 \\ (-) \quad (+) \quad (-) \\ \hline 0 = -1 \\ \hline \end{array}$$

Here we arrive at a contradiction as $0 \neq 1$

This mean that the system is inconsistent and has no solution.

Example 3.7

Solve $\frac{x}{2} - 1 = \frac{y}{6} + 1 = \frac{x}{7} + 2$; $\frac{y}{3} + \frac{z}{2} = 13$

Sol Consider, $\frac{x}{2} - 1 = \frac{y}{6} + 1$

$$\frac{x-2}{2} = \frac{y+6}{6}$$

$$6(x-2) = 2(y+6)$$

$$6x - 12 = 2y + 12$$

$$6x - 2y - 12 - 12 = 0$$

$$6x - 2y = 24$$

$$3x - y = 12 \longrightarrow (1)$$

Considering, $\frac{x}{2} - 1 = \frac{z}{7} + 2$

$$\frac{x-2}{2} = \frac{z+14}{7}$$

$$7(x-2) = 2(z+14)$$

$$7x - 14 = 2z + 28$$

$$7x - 2z = 28 + 14$$

$$7x - 2z = \cancel{14}\ 42 \longrightarrow (2)$$

Also $\frac{y}{3} + \frac{z}{2} = 13$

$$\frac{2y+3z}{6} = 13$$

$$2y + 3z = 13 \times 6$$

$$2y + 3z = 78 \longrightarrow (3)$$

Eliminating z from (2) and (3)

$(2) \times 3 \Rightarrow \quad 21x - 6z = 126$

$(3) \times 2 \Rightarrow \quad \cancel{4}\ 4y + 6z = 156$

$$21x + 4y = 282$$

$(1) \times 4 \Rightarrow \quad 12x - 4y = 48$

$$33x = 330$$

$$x = \frac{330}{33}$$

$$\boxed{x = 10}$$

Substitute $x = 10$ in (1)

$$3(10) - y = 12$$

$$30 - y = 12$$

$$-y = 12 - 30$$

$$-y = -18$$

$$\boxed{y = 18}$$

Substitute $x = 10$ in (2)

$$7(10) - 2z = 42$$

$$70 - 2z = 42$$

$$-2z = 42 - 70$$

$$-2z = -28$$

$$z = \frac{28}{2}$$

$$\boxed{z = 14}$$

$\therefore\ x = 10,\ y = 18,\ z = 14.$

Example 3.8

Solve : $\frac{1}{2x} + \frac{1}{4y} - \frac{1}{3z} = \frac{1}{4}$; $\frac{1}{x} = \frac{1}{3y}$; $\frac{1}{x} - \frac{1}{5y} + \frac{4}{z} = 2\frac{2}{15}$

Sol Let $\frac{1}{x} = a$, $\frac{1}{y} = b$; $\frac{1}{z} = c$

The given equation,

$$\frac{a}{2} + \frac{b}{4} - \frac{c}{3} = \frac{1}{4}$$

$$a = \frac{b}{3}$$

$$a - \frac{b}{5} + 4c = \frac{32}{15}$$

By simplifying,

$$\frac{6a + 3b - 4c}{12} = \frac{1}{4}$$

$$6a + 3b - 4c = 12/4$$

$$6a + 3b - 4c = 3 \longrightarrow (1)$$

$$3a = b \longrightarrow (2)$$

$$15(a - b/5 + 4c) = 32$$

$$15a - \frac{15b}{5} + 60c = 32$$

$$15a - 3b + 60c = 32 \longrightarrow (3)$$

Substitute ② in ①

$$6a + 3(3a) - 4c = 3$$

$$6a + 9a - 4c = 3$$

$$15a - 4c = 3 \longrightarrow ④$$

Substitute ② in ③

$$15a - 3(3a) + 60c = 32$$

$$15a - 9a + 60c = 32$$

$$6a + 60c = 32$$

$$3a + 30c = 16 \longrightarrow ⑤$$

solve ④ & ⑤

$$④ \Rightarrow \quad 15a - 4c = 3$$

$$⑤ \times 5 \Rightarrow \quad 15a + 150c = 80$$

$$\ominus \quad \ominus \quad \ominus$$

$$-154c = -77$$

$$c = \frac{77}{154}$$

$$c = 1/2$$

Substitute $c = 1/2$ in ④

$$15a - 4(1/2) = 3$$

$$15a - 2 = 3$$

$$15a = 3 + 2$$

$$15a = 5$$

$$a = 5/15$$

$$\boxed{a = 1/3}$$

$a = 1/3$ in ②

$$3(1/3) = b$$

$$\boxed{b = 1}$$

$$\therefore x = \frac{1}{a} = 3 \; ; \; y = \frac{1}{b} = 1 \; ; \; z = \frac{1}{c} = 2$$

$$\therefore x = 3, \; y = 1, \; z = 2$$

Example 3.9

The sum of thrice the first number, second number, and twice the third number is 5. If thrice the second number is subtracted from the sum of first number and thrice the third we get 2. If the third number is subtracted from the sum of twice the first third is second, we get 1. Find the numbers.

Sol

Let the three numbers be x, y, z

From the given data we get the following equations.

$$3x + y + 2z = 5 \longrightarrow (1)$$

$$x + 3y + 3z = 2 \longrightarrow (2)$$

$$2x + 3y - z = 1 \longrightarrow (3)$$

$(1) \times 1 \Rightarrow \quad 3x + y + 2z = 5$

$(2) \times 3 \Rightarrow \quad 3x - 9y + 9z = 6$

$\quad (-) \quad (+) \quad (-) \quad (-) \quad (-)$

$$10y - 7z = -1 \longrightarrow (4)$$

$(1) \times 2 \Rightarrow \quad 6x + 2y + 4z = 10$

$(3) \times 3 \Rightarrow \quad 6x + 9y - 3z = 3$

$\quad (-) \quad (-) \quad (+) \quad (-)$

$$-7y + 7z = 7 \longrightarrow (5)$$

Adding (4) & (5)

$$10y - 7z = -1$$
$$-7y + 7z = 7$$
$$3y = 6$$
$$y = 6/3$$
$$\boxed{y = 2}$$

Substitute $y = 2$ in (5)

$$-7(2) + 7z = 7$$
$$-14 + 7z = 7$$
$$7z = 7 + 14$$
$$7z = 21$$

$$z = \frac{21}{7}$$

$$\boxed{z = 3}$$

substitute $y = 2$, $z = 3$ in ①

$$3x + 2 + 2(3) = 5$$

$$3x + 2 + 6 = 5$$

$$3x + 8 = 5$$

$$3x = 5 - 8$$

$$3x = -3$$

$$x = -3/3$$

$$\boxed{x = -1}$$

$\therefore$ $x = -1$, $y = 2$, $z = 3$

Example 3.10

Find the GCD of the polynomial $x^3 + x^2 - x + 2$ and $2x^3 - 5x^2 + 5x - 3$.

Sol

Let $f(x) = 2x^3 - 5x^2 + 5x - 3$ and

$g(x) = x^3 + x^2 - x + 2$

$$\begin{array}{r|l} & 2 \\ x^3 + x^2 - x + 2 & 2x^3 - 5x^2 + 5x - 3 \\ & 2x^3 + 2x^2 - 2x + 4 \quad (-) \\ \hline & -7x^2 + 7x - 7 \end{array}$$

$-7(x^2 - x + 1) \neq 0$, -7 is not divisor of $g(x)$

Now dividing $g(x) = x^3 + x^2 - x + 2$ by the new remainder $x^2 - x + 1$ (leaving constant factor).

$$
\begin{array}{r|l}
 & x+2 \\
\hline
x^2-x+1 & x^3+x^2-x+2 \\
 & x^3-x^2+x \quad (-) \\
\hline
 & 2x^2-2x+2 \\
 & 2x^2-2x+2 \\
\hline
 & 0
\end{array}
$$

Here, we get zero remainder

$\therefore$ GCD $(2x^3-5x^2+5x-3,\ x^3+x^2-x+2) = x^2-x+1$

Example 3.11

Find the GCD of $6x^3-30x^2+60x-48$ and $3x^3-12x^2+21x-18$

Sol Let $f(x) = 6x^3-30x^2+60x-48 = 6(x^3-5x^2+10x-8)$

$g(x) = 3x^3-12x^2+21x-18 = 3(x^3-4x^2+7x-6)$

Find GCD of $x^3-5x^2+10x-8$ and x^3-4x^2+7x-6

$$
\begin{array}{r|l}
 & 1 \\
\hline
x^3-5x^2+10x-8 & x^3-4x^2+7x-6 \\
 & x^3-5x^2+10x-8 \quad (-) \\
\hline
 & x^2-3x+2
\end{array}
$$

$$
\begin{array}{r|l}
 & x-2 \\
\hline
x^2-3x+2 & x^3-5x^2+10x-8 \\
 & x^3-3x^2+2x \quad (-) \\
\hline
 & -2x^2+8x-8 \\
 & -2x^2+6x-4 \quad (-) \\
\hline
 & 2x-4 \\
 & = 2(x-2)
\end{array}
$$

$$
\begin{array}{r|l}
 & x-1 \\
\hline
x-2 & x^2-3x+2 \\
 & x^2-2x \quad (-) \\
\hline
 & -x+2 \\
 & -x+2 \quad (-) \\
\hline
 & 0
\end{array}
$$

Here we get zero remainder

GCD of leading coefficients 3 and 6 is 3

Thus, GCD $[6x^3-30x^2+60x-48,\ 3x^3-12x^2+21x-18]$

$= 3(x-2)$

Example 3.12

Find the LCM of the following

(i) $8x^4y^2,\ 48x^2y^4$ (ii) $5x-10,\ 5x^2-20$

(iii) $x^4-1,\ x^2-2x+1$ (iv) $x^3-27,\ (x-3)^2,\ x^2-9$

Sol (i) $8x^4y^2,\ 48x^2y^4$

Find LCM of the numerical coefficient.

$$\begin{array}{r|l} 2 & 8,\ 48 \\ \hline 2 & 4,\ 24 \\ \hline 2 & 2,\ 12 \\ \hline & 1,\ 6 \end{array}$$

$LCM\ (8, 48) = 2\times2\times2\times1\times6$

$= 48$

Find LCM of the terms involving variables.

$LCM\ (x^4y^2,\ x^2y^4) = x^4y^4$

Find LCM of the Given expressions

The LCM of the given expression is product of the LCM of the numerical coefficient and the LCM of the term with variables.

$\therefore LCM\ (8x^4y^2,\ 48x^2y^4) = 48x^4y^4$

(ii) $(5x-10),\ (5x^2-20)$

$5x-10 = 5(x-2)$

$5x^2-20 = 5(x^2-4)$

$= 5(x^2-2^2)$

$= 5(x+2)(x-2)$

$\therefore LCM\ [(5x-10),\ (5x^2-20)] = 5(x+2)(x-2)$

(iii) $(x^4-1),\ x^2-2x+1$

$x^4-1 = (x^2)^2-1^2$

$= (x^2+1)(x^2-1)$

$= (x^2+1)(x+1)(x-1)$

$x^2-2x+1 = (x-1)^2$

$\therefore$ LCM $[(x^4-1), (x^2-2x+1)] = (x^2+1)(x+1)(x-1)^2$

(iv) $x^3-27, (x-3)^2, x^2-9$

$x^3-27 = x^3-9^3$

$= (x-3)(x^2+3x+9)$

$(x-3)^2 = (x-3)^2$

$x^2-9 = (x^2-3^2)$

$= (x+3)(x-3)$

$\therefore$ LCM $[x^3-27, (x-3)^2, x^2-9] = (x+3)(x-3)^2(x^2+3x+9)$

Example 3.13

Reduce the rational expression to its lowest form.

(i) $\frac{x-3}{x^2-9}$ (ii) $\frac{x^2-16}{x^2+8x+16}$

Sol

(i) $\frac{x-3}{x^2-9} = \frac{x-3}{(x-3)(x+3)} = \frac{1}{(x+3)}$

(ii) $\frac{x^2-16}{x^2+8x+16} = \frac{x^2-4^2}{(x+4)(x+4)} = \frac{(x+4)(x-4)}{(x+4)(x+4)} = \frac{x-4}{x+4}$

Example 3.14

Find the excluded values of the following expressions.

(i) $\frac{x+10}{8x}$ (ii) $\frac{7p+2}{8p^2+13p+5}$ (iii) $\frac{x}{x^2+1}$

Sol

(i) $\frac{x+10}{8x}$

The expression $\frac{x+10}{8x}$ is undefined when $8x=0$ or $x=0$. Hence the excluded value is 0.

(ii) $\dfrac{7p+2}{8p^2+13p+5}$

The expression $\dfrac{7p+2}{8p^2+13p+5}$ is undefined when

40

8 13 5

$$8p^2+13p+5 = 0$$

$$8p^2+8p+5p+5 = 0$$

$$8p(p+1)+5(p+1) = 0$$

$$(8p+5)(p+1) = 0$$

$$p = -\frac{5}{8}, \quad p = -1$$

$\therefore$ The excluded values are $\frac{-5}{8}$ and -1

(iii) $\dfrac{x}{x^2+1}$

Here $x^2 \geq 0$ for all x.

$\therefore x^2+1 \geq 0+1 = 1$.

Hence, $x^2+1 \neq 0$ for any x

$\therefore$ There can be no real excluded values for the given rational expression $\dfrac{x}{x^2+1}$

Example 3.15

(i) Multiply $\dfrac{x^3}{9y^2}$ by $\dfrac{27y}{x^5}$ (ii) Multiply $\dfrac{x^4b^2}{x-1}$ by $\dfrac{x^2-1}{a^4b^3}$

Sol (i) $\dfrac{x^3}{9y^2} \times \dfrac{27y}{x^5} = \dfrac{3x^3}{9yx^5} = \dfrac{3}{9yx^2}$

(ii) $\dfrac{x^4b^2}{x-1} \times \dfrac{x^2-1}{a^4b^3} = \dfrac{x^4b^2}{x-1} \times \dfrac{(x+1)(x-1)}{a^4b^3}$

$$= \frac{x^4(x+1)}{a^4b}$$

Example 3.16

Find (i) $\frac{14x^4}{y} \div \frac{7x}{3y^4}$ (ii) $\frac{x^2-16}{x+4} \div \frac{x-4}{x+4}$ ~~(iii)~~ (15)

(iii) $\frac{16x^2-2x-3}{3x^2-2x-1} \div \frac{8x^2+11x+3}{3x^2-11x-4}$

Sol

(i) $\frac{14x^4}{y} \div \frac{7x}{3y^4} = \frac{14x^4}{y} \times \frac{3y^4}{7x} = 6x^3y^3$

(ii) $\frac{x^2-16}{x+4} \div \frac{x-4}{x+4} = \frac{x^2-4^2}{x+4} \div \frac{x-4}{x+4}$

$= \frac{(x+4)(x-4)}{x+4} \times \frac{x+4}{(x-4)}$

$= x+4$

(iii) $\frac{16x^2-2x-3}{3x^2-2x-1} \div \frac{8x^2+11x+3}{3x^2-11x-4}$

$= \frac{16x^2-2x-3}{3x^2-2x-1} \times \frac{3x^2-11x-4}{8x^2+11x+3}$

$= \frac{16x^2+6x-8x-3}{3x^2+x-3x-1} \times \frac{3x^2+x-12x-4}{8x^2+8x+3x+3}$

$= \frac{2x(8x+3)-1(8x+3)}{x(3x+1)-1(3x+1)} \times \frac{3x(x+1)-4(3x+1)}{8x(x+1)+3(x+1)}$

$= \frac{(2x-1)(8x+3)}{(x-1)(3x+1)} \times \frac{(x-4)(3x+1)}{(8x+3)(x+1)}$

$= \frac{(2x-1)(x-4)}{(x-1)(x+1)} = \frac{2x^2-8x-x+4}{x^2-1}$

$= \frac{2x^2-9x+4}{x^2-1}$

Example 3.17

Find $\dfrac{x^2+20x+36}{x^2-3x-28} - \dfrac{x^2+12x+4}{x^2-3x-28}$

Sol

$$\frac{x^2+20x+36}{x^2-3x-28} - \frac{x^2+12x+4}{x^2-3x-28}$$

$$= \frac{(x^2+20x+36)-(x^2+12x+4)}{x^2-3x-28}$$

$$= \frac{x^2+20x+36-x^2-12x-4}{x^2-3x-28}$$

$$= \frac{8x+32}{x^2-3x-28}$$

-28

$-7 \quad -3 \quad 4$

$$= \frac{8x+32}{x^2-7x+4x-28}$$

$$= \frac{8(x+4)}{x(x-7)+4(x-7)}$$

$$= \frac{8(x+4)}{(x-7)(x+4)} = \frac{8}{x-7}$$

Example 3.18

Simplify $\dfrac{1}{x^2-5x+6} + \dfrac{1}{x^2-3x+2} - \dfrac{1}{x^2-8x+15}$

Sol

$$\frac{1}{x^2-5x+6} + \frac{1}{x^2-3x+2} - \frac{1}{x^2-8x+15}$$

6: -3, -2 (-5) 2: -1, -2 (-3) 15: -5, -3 (-8)

$$= \frac{1}{x^2-3x-2x+6} + \frac{1}{x^2-x-2x+2} - \frac{1}{x^2-5x-3x+15}$$

$$= \frac{1}{x(x-3)-2(x-3)} + \frac{1}{x(x-1)-2(x-1)} - \frac{1}{x(x-5)-3(x-5)}$$

$$= \frac{1}{(x-3)(x-2)} + \frac{1}{(x-1)(x-2)} - \frac{1}{(x-3)(x-5)}$$

$$= \frac{(x-1)(x-5) + (x-3)(x-5) - (x-1)(x-2)}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{x^2-5x-x+5 + x^2-5x-3x+15 - (x^2-x-2x+2)}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{x^2-6x+5+x^2-8x+15-x^2+3x-2}{(x-1)(x-2)(x-3)(x-5)}$$

(18)

$$= \frac{x^2-14x+3x+20-2}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{x^2-11x+18}{(x-1)(x-2)(x-3)(x-5)}$$

18

-11

$\frac{-9}{x}$ $\frac{-2}{x}$

$$= \frac{(x-9)(x-2)}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{x-9}{(x-1)(x-3)(x-5)}$$

Example 3.19

Find the square root of the following expressions.

(i) $256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}$

(ii) $\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}$.

Sol

(i) $\sqrt{256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}}$

$$= 16\left|(x-a)^4(x-b)^2(x-c)^8(x-d)^{10}\right|$$

(ii) $$\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}} = \frac{12}{9}\left|\frac{a^4b^6c^8}{f^6g^2h^7}\right|$$

$$= \frac{4}{3}\left|\frac{a^4b^6c^8}{f^6g^2h^7}\right|$$

Example 3.20

Find the square root of the following expressions.

(i) $16x^2+9y^2-24xy+24x-18y+9$

(ii) $(6x^2+x-1)(3x^2+2x-1)(2x^2+3x+1)$.

(iii) $[\sqrt{15}x^2+(\sqrt{3}+\sqrt{10})x+\sqrt{2}][\sqrt{5}x^2+(2\sqrt{5}+1)x+2]$

$[\sqrt{3}x^2+(\sqrt{2}+2\sqrt{3})x+2\sqrt{2}]$

Sol

(i) $16x^2+9y^2-24xy+24x-18y+9$

$= \sqrt{16x^2+9y^2-24xy+24x-18y+9}$

$= \sqrt{(4x)^2+(3y)^2-2(4x)(-3y)+2(4x)(3)-2(-3y)(3)}$

$= \sqrt{(4x-3y+3)^2} = |4x-3y+3|$

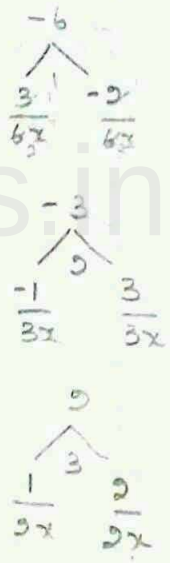
(ii) $\sqrt{(6x^2+x-1)(3x^2+2x-1)(2x^2+3x+1)}$

$= \sqrt{(2x+1)(3x-1)(3x-1)(x+1)(2x+1)(x+1)}$

$= \sqrt{(2x+1)^2(3x-1)^2(x+1)^2}$

$= \sqrt{(2x+1)^2(3x-1)^2(x+1)^2}$

$= |(2x+1)(3x-1)(x+1)|$



(iii) Let us factorize the polynomial

$\sqrt{15}x^2+(\sqrt{3}+\sqrt{10})x+\sqrt{2} = \sqrt{15}x^2+\sqrt{3}x+\sqrt{10}x+\sqrt{2}$

$= \sqrt{3}x(\sqrt{5}x+1)+\sqrt{2}(\sqrt{5}x+1)$

$= (\sqrt{3}x+\sqrt{2})(\sqrt{5}x+1)$

$\sqrt{5}x^2+(2\sqrt{5}+1)x+2 = \sqrt{5}x^2+2\sqrt{5}x+x+2$

$= \sqrt{5}x(x+2)+1(x+2)$

$\sqrt{3}x^2+(\sqrt{2}+2\sqrt{3})x+2\sqrt{2} = \sqrt{3}x^2+\sqrt{2}x+2\sqrt{3}x+2\sqrt{2}$

$$= x(\sqrt{3}x + \sqrt{2}) + 2(\sqrt{3}x + \sqrt{2})$$

$$= (x+2)(\sqrt{3}x + \sqrt{2})$$

$$\sqrt{\left[\sqrt{15}x^2 + (\sqrt{3}+\sqrt{10})x + \sqrt{2}\right]\left[\sqrt{5}x^2 + (2\sqrt{5}+1)x + 2\right]\left[\sqrt{3}x^2 + (\sqrt{2}+2\sqrt{3})x + 2\sqrt{2}\right]}$$

$$= \sqrt{(\sqrt{5}x+1)(\sqrt{3}x+\sqrt{2})(\sqrt{5}x+1)(x+2)(\sqrt{3}x+\sqrt{2})(x+2)}$$

$$= \sqrt{(\sqrt{5}x+1)^2(\sqrt{3}x+\sqrt{2})^2(x+2)^2}$$

$$= \left|(\sqrt{5}x+1)(\sqrt{3}x+\sqrt{2})(x+2)\right|$$

Example 3.21

Find the square root of $64x^4 - 16x^3 + 17x^2 - 2x + 1$

Sol

$$
\begin{array}{r|l}
 & 8x^2 - x + 1 \\
\hline
8x^2 & 64x^4 - 16x^3 + 17x^2 - 2x + 1 \\
 & 64x^4 \quad (-) \\
\hline
16x^2 - x & -16x^3 + 17x^2 \\
 & -16x^3 + x^2 \quad (-) \\
\hline
16x^2 - 2x + 1 & 16x^2 - 2x + 1 \\
 & 16x^2 - 2x + 1 \\
\hline
 & 0
\end{array}
$$

$$\therefore \sqrt{64x^4 - 16x^3 + 17x^2 - 2x + 1} = |8x^2 - x + 1|$$

Example 3.22

If $9x^4 + 15x^3 + 28x^2 + ax + b$ is a perfect square find the values of a and b.

Sol

$$
\begin{array}{r|l}
 & 3x^2+2x+4 \\
\hline
3x^2 & 9x^4+12x^3+28x^2+ax+b \\
 & 9x^4 \qquad\qquad\qquad\qquad (-) \\
\hline
6x^2+2x & 12x^3+28x^2 \\
 & 12x^3+4x^2 \qquad\qquad (-) \\
\hline
6x^2+4x+4 & 24x^2+ax+b \\
 & 24x^2+16x+16 \quad (-) \\
\hline
 & 0
\end{array}
$$

Because the given polynomial is a perfect square

$a-16=0, \quad b-16=0$

$\therefore \; a=16, \quad b=16.$

Example 3.23

Find the zeroes of the following quadratic expressions $x^2+8x+12$.

Sol

Let $P(x) = x^2+8x+12 = (x+2)(x+6)$

$P(-2) = (-2)^2+8(-2)+12$

$= 4-16+12$

$= 16-16 = 0$

$P(-6) = (-6)^2+8(-6)+12$

$= 36-48+12$

$= -12+12 = 0$

$\therefore$ -2 and -6 are zeroes of $P(x) = x^2+8x+12$.

Example 3.24

Write down the quadratic equation in general form for which sum and product of the roots are given below.

(i) 9, 14 (ii) $-\frac{7}{2}, \frac{5}{2}$ (iii) $-\frac{3}{5}, -\frac{1}{2}$

80) (i) General form of the quadratic equation when the roots are given is

$x^2 -$ (sum of the roots) x + product of the roots $= 0$

$x^2 - 9x + 14 = 0$

(ii) Sum of the roots $= -7/2$

Product of the roots $= 5/2$

General form: $x^2 -$ (sum of the roots) x + product of the roots $= 0$

$$x^2 - \left(-\frac{7}{2}\right)x + \frac{5}{2} = 0$$

$$x^2 + \frac{7}{2}x + \frac{5}{2} = 0$$

$$\frac{2x^2 + 7x + 5}{2} = 0$$

$$2x^2 + 7x + 5 = 0$$

(iii) Sum of the roots $= -3/5$

Product of the roots $= -1/2$

$x^2 -$ (sum of the roots) x + product of the roots $= 0$

$$x^2 - \left(-3/5\right)x + \left(-1/2\right) = 0$$

$$x^2 + \frac{3}{5}x - \frac{1}{2} = 0$$

$$\frac{10x^2 + 6x - 5x}{10} = 0$$

$$10x^2 + 6x - 5x = 0$$

Example 3.25

Find the sum and product of the roots for each of the following quadratic equations

(i) $x^2+8x-65=0$ (ii) $2x^2+5x+7=0$ (iii) $kx^2-k^2x-2k^3=0$

Sol

Let α and β be the roots of given quadratic equation.

(i) $x^2+8x-65=0$

$a=1,\ b=8,\ c=-65$

sum of the roots $\alpha+\beta=-\frac{b}{a}=-\frac{8}{1}=-8$

product of the roots $\alpha\beta=\frac{c}{a}=\frac{-65}{1}=-65$

$\therefore\ \alpha+\beta=-8,\ \alpha\beta=-65.$

(ii) $2x^2+5x+7=0$

$a=2,\ b=5,\ c=7$

sum of the roots $\alpha+\beta=\frac{-b}{a}=-\frac{5}{2}$

product of the roots $\alpha\beta=\frac{c}{a}=\frac{7}{2}$

$\alpha+\beta=-5/2;\ \alpha\beta=7/2.$

(iii) $kx^2-k^2x-2k^3=0$

$a=k,\ b=-k^2,\ c=-2k^3$

sum of the roots $\alpha+\beta=-\frac{b}{a}=\frac{k^2}{k}=k$

Product of the roots $\alpha\beta=\frac{c}{a}=\frac{-2k^3}{k}=-2k^2$

$\alpha+\beta=k,\ \alpha\beta=-2k^2$

Example 3.26

Solve $2x^2-2\sqrt{6}x+3=0$

Sol

$2x^2-2\sqrt{6}x+3=2x^2-\sqrt{6}x-\sqrt{6}x+3$ (By spliting).

$$= \sqrt{2}x(\sqrt{2}x - \sqrt{3}) - \sqrt{3}(\sqrt{2}x - \sqrt{3})$$

$$= (\sqrt{2}x - \sqrt{3})(\sqrt{2}x - \sqrt{3})$$

Equate the factor to zero

$$(\sqrt{2}x - \sqrt{3})(\sqrt{2}x - \sqrt{3}) = 0$$

$$(\sqrt{2}x - \sqrt{3})^2 = 0$$

$$\sqrt{2}x - \sqrt{3} = 0$$

$$\sqrt{2}x = \sqrt{3}$$

$$x = \frac{\sqrt{3}}{\sqrt{2}}$$

$\therefore$ The solution is $x = \frac{\sqrt{3}}{\sqrt{2}}$

Example 3.27

Solve $2m^2 + 19m + 30 = 0$

Sol

$$2m^2 + 19m + 30 = 2m^2 + 4m + 15m + 30$$

$$= 2m(m+2) + 15(m+2)$$

$$= (2m+15)(m+2)$$

Equating the factor to zero

$$(2m+15)(m+2) = 0$$

$m + 2 = 0$	$2m + 15 = 0$
$m = -2$	$2m = -15$
	$m = -15/2$

$\therefore$ The roots are $-2, -15/2$

Example 3.28

Solve $x^4 - 13x^2 + 42 = 0$

Sol

Let $x^2 = a$

$$(x^2)^2 - 13x^2 + 42 = 0$$

$$a^2 - 13a + 42 = 0$$

$$a^2 - 13a + 42 = a^2 - 7a - 6a + 42$$

$$= (a-7)(a-6)$$

$$(a-7)(a-6) = 0$$

$$a = 7 ; \quad a = 6$$

Since $a = x^2$, $x^2 = 7$ then $x = \pm\sqrt{7}$

$x^2 = 6$ then $x = \pm\sqrt{6}$

$\therefore$ The roots are $x = \pm\sqrt{7}, \pm\sqrt{6}$

Example 3.29

Solve $\dfrac{x}{x-1} + \dfrac{x-1}{x} = 2\dfrac{1}{2}$

Sol

let $y = \dfrac{x}{x-1}$; $\dfrac{1}{y} = \dfrac{x-1}{x}$

$$\therefore \quad y + \frac{1}{y} = 2\frac{1}{2} \Rightarrow y + \frac{1}{y} = \frac{5}{2}$$

$$\Rightarrow \frac{y^2+1}{y} = \frac{5}{2}$$

$$\Rightarrow 2(y^2+1) = 5y$$

$$\Rightarrow 2y^2 + 2 - 5y = 0$$

$$\Rightarrow 2y^2 - 5y + 2 = 0$$

$$(2y-1)(y-2) = 0$$

$$y = 1/2 , \quad y = 2$$

$\dfrac{x}{x-1} = \dfrac{1}{2}$; $2x = x - 1 \Rightarrow 2x - x = -1$

$x = -1$

$\dfrac{x}{x-1} = 2$; $x = 2x - 2 \Rightarrow x = 2$

$\therefore$ The roots are $x = -1, 2$.

Example 3.30

Solve $x^2 - 3x - 2 = 0$

Sol

$$x^2 - 3x - 2 = 0$$

$$x^2 - 3x = 2$$

(Add $[1/2 \text{ (coefficient of } x)]^2$ to both side

$$x^2 - 3x + \left(\frac{3}{2}\right)^2 = 2 + \left(\frac{3}{2}\right)^2$$

$$(x - 3/2)^2 = 2 + \frac{9}{4}$$

$$(x - 3/2)^2 = \frac{8+9}{4}$$

$$(x - 3/2)^2 = 17/4$$

Take square root on both side

$$x - 3/2 = \pm\sqrt{\frac{17}{4}}$$

$$x = \frac{3}{2} + \frac{\sqrt{17}}{2} \text{ or } x = \frac{3}{2} - \frac{\sqrt{17}}{2}$$

$$\therefore x = \frac{3+\sqrt{17}}{2}, \frac{3-\sqrt{17}}{2}.$$

Example 3.31

Solve $2x^2 - x - 1 = 0$

Sol

$$2x^2 - x - 1 = 0$$

$\div$ by 2

$$x^2 - \frac{x}{2} - \frac{1}{2} = 0$$

$$x^2 - \frac{x}{2} = \frac{1}{2}$$

Add $[1/2 \text{ (coefficient of } x)]^2$ on both side

$$x^2 - \frac{x}{2} + \left(\frac{1}{4}\right)^2 = \frac{x^2}{2} + \left(\frac{1}{4}\right)^2$$

$$(x - 1/4)^2 = \frac{1}{2} + \frac{1}{16}$$

$$(x - 1/4)^2 = \frac{18}{32}$$

$$(x - 1/4)^2 = 9/16$$

$$(x - 1/4)^2 = (3/4)^2$$

$$x - 1/4 = \pm 3/4$$

$$x = 3/4 + 1/4 = 4/4 = 1 \; ; \; x = -3/4 + 1/4 = \frac{-2}{4} = \frac{-1}{2}$$

$$\therefore x = 1, -1/2$$

Example 3.39.

solve $x^2 + 2x - 2 = 0$ by formula method.

Sol

Compare $x^2 + 2x - 2 = 0$ with the standard form $ax^2 + bx + c = 0$

$$a = 1, \; b = 2, \; c = -2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

substitute a, b, c values

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4 + 8}}{2} = \frac{-2 \pm \sqrt{12}}{2} = \frac{-2 \pm 2\sqrt{3}}{2}$$

$$= \frac{2(-1 \pm \sqrt{3})}{2} = -1 \pm \sqrt{3}$$

$$\therefore x = -1 + \sqrt{3}, \; -1 - \sqrt{3}$$

Example 3.33

solve $2x^2 - 3x - 3 = 0$ by formula method.

Sol

Compare $2x^2 - 3x - 3 = 0$ with $ax^2 + bx + c = 0$

$a = 2,\ b = -3,\ c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-3)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{9 + 24}}{4}$$

$$= \frac{3 \pm \sqrt{33}}{4}$$

$$\therefore\ x = \frac{3 + \sqrt{33}}{4},\ \frac{3 - \sqrt{33}}{4}$$

Example 3.34.

Solve $3p^2 + 2\sqrt{5}p - 5 = 0$ by formula method.

Sol

Compare $3p^2 + 2\sqrt{5}p - 5 = 0$ with $ax^2 + bx + c = 0$

$a = 3,\ b = 2\sqrt{5},\ c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2\sqrt{5} \pm \sqrt{(2\sqrt{5})^2 - 4(3)(-5)}}{2(3)}$$

$$x = \frac{-2\sqrt{5} \pm \sqrt{20 + 60}}{6}$$

$$x = \frac{-2\sqrt{5} \pm \sqrt{80}}{6}$$

$$x = \frac{-2\sqrt{5} \pm \sqrt{16 \times 5}}{6}$$

$$x = \frac{-2\sqrt{5} \pm 4\sqrt{5}}{6}$$

$$x = \frac{2(-\sqrt{5} \pm 2\sqrt{5})}{6}$$

$$x = \frac{-\sqrt{5} \pm 2\sqrt{5}}{3}$$

$$\therefore x = \frac{-\sqrt{5} + 2\sqrt{5}}{3} = \frac{\sqrt{5}}{3}$$

$$x = \frac{-\sqrt{5} - 2\sqrt{5}}{3} = -\frac{3\sqrt{5}}{3} = -\sqrt{5}$$

Example 3.35

Solve $pqx^2 - (p+q)^2 x + (p+q)^2 = 0$

Sol

compare coefficients with $ax^2 + bx + c = 0$

$a = pq$, $b = -(p+q)^2$, $c = (p+q)^2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{(p+q)^2 \pm \sqrt{(-(p+q)^2)^2 - 4(pq)(p+q)^2}}{2pq}$$

$$= \frac{(p+q)^2 \pm \sqrt{(p+q)^4 - 4(pq)(p+q)^2}}{2pq}$$

$$= \frac{(p+q)^2 \pm \sqrt{(p+q)^2[(p+q)^2 - 4pq]}}{2pq}$$

$$= \frac{(p+q)^2 \pm \sqrt{(p+q)^2(p^2+q^2+2pq-4pq}}{2pq}$$

$$= \frac{(p+q)^2 \pm \sqrt{(p+q)^2(p^2+q^2-2pq)}}{2pq}$$

$$= \frac{(p+q)^2 \pm \sqrt{(p+q)^2(p-q)^2}}{2pq}$$

$$= \frac{(p+q)^2 \pm (p+q)(p-q)}{2pq}$$

$$= \frac{p+q\,[(p+q) \pm (p-q)]}{2pq}$$

$$x = \frac{p+q\,[p+q+p-q]}{2pq} = \frac{p+q}{2pq} \times 2p$$

$$= \frac{p+q}{q}$$

$$x = \frac{p+q\,[p+q-p+q]}{2pq} = \frac{p+q}{2pq} \times 2q$$

$$= \frac{p+q}{p}$$

$$\therefore x = \frac{p+q}{q}, \frac{p+q}{p}$$

Example 3.36

The product of Kumaran's age (in years) two years ago and his age four years from now is one more than twice his present age. What is his present age?

Sol Let the present age of Kumaran be x years

two years ago, his age $= (x-2)$ years

Four years from now, his age $= (x+4)$ years

Given;

$$(x-2)(x+4) = 1+2x$$

$$x^2+4x-2x-8 = 1+2x$$

$$x^2+2x-8 = 1+2x$$

$$x^2-8+2x-1-2x = 0$$

$$x^2-9 = 0$$

$$x^2 = 9$$

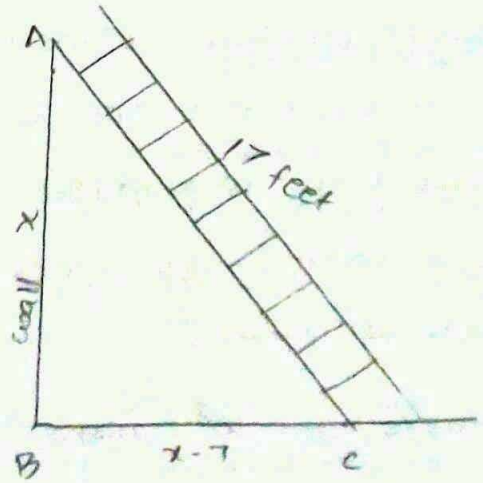
$$x = \pm 3$$

$\therefore x = 3$ (-3 age cannot be negative).

Kumaran's present age is 3 years.

Example 3.37

A ladder 17 feet long is leaning against a wall. If the ladder, vertical wall and the floor from the bottom of the wall to the ladder form a right triangle, find the height of the wall where the top of the ladder meets If the distance between bottom of the wall to bottom of the ladder is 7 feet less than the height of the wall?



Sol

Let the height of the wall AB = x feet

As per the given data BC $= (x-7)$ feet

In the right triangle ABC,

$AC = 17$ ft, $BC = (x-7)$ feet

By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$(17)^2 = x^2 + (x-7)^2$$

$$289 = x^2 + x^2 - 14x + 49$$

$$289 = 2x^2 - 14x + 49$$

$$289 - 49 = 2x^2 - 14x$$

$$-2x^2 + 14x + 240 = 0$$

$$x^2 - 7x - 120 = 0$$

$$x^2 - 15x + 8x - 120 = 0$$

$$x(x-15) + 8(x-15) = 0$$

$$(x+8)(x-15) = 0$$

$$x = 15, \; x = -8$$

$\therefore$ Height of the wall AB = 15 feet

(-8, height cannot be negative)

Example 3.38

A flock of swans contained x^2 members. As the clouds gathered, $10x$ went to a lake and one-eighth of the member flew away to a garden. the remaining three pairs played about in the water. How many swans were there in total?

Sol

As given there are x^2 swans.

The given data $x^2 - 10x - \frac{1}{8}x^2 = 6$

$$\frac{8x^2 - 80x - x^2}{8} = 6$$

$$8x^2 - 80x - x^2 = 48$$

$$7x^2 - 80x - 48 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{80 \pm \sqrt{(80)^2 - 4(7)(-48)}}{2(7)}$$

$$= \frac{80 \pm \sqrt{6400 + 1344}}{14}$$

$$= \frac{80 \pm \sqrt{7744}}{14} = \frac{80 \pm 88}{14} = \frac{2(40 \pm 44)}{14}$$

$$x = \frac{40 \pm 44}{7}$$

$$x = \frac{40 + 44}{7} = \frac{84}{7} = 12$$

$$x = \frac{40 - 44}{7} = \frac{-4}{7}$$

$$x = 12, -\frac{4}{7}$$

$x = -\frac{4}{7}$ (swans cannot be negative)

$\therefore x = 12$

$\therefore$ Total number of swans $x^2 = 12^2$

$$x^2 = 144.$$

Example 3.39:

A passenger train takes 1 hr more than an express train to travel distance of 240 km from chennai to virudhachalam

The speed of passenger train is less than that of an express train by 20 km per hour. Find the average speed of both the trains. (94)

Sol

Let the average speed of passenger train be x km/hr.

The average speed of express train will be $(x+20)$ km/hr.

Time taken by the passenger train to cover distance of 240 km $= \frac{240}{x}$ hr.

Time taken by express train to cover distance of 240 km $= \frac{240}{x+20}$ hr.

Given

$$\frac{240}{x} = \frac{240}{x+20} + 1$$

$$\frac{240}{x} - \frac{240}{x+20} = 1$$

$$240\left[\frac{1}{x} - \frac{1}{x+20}\right] = 1$$

$$240\left[\frac{x+20-x}{x(x+20)}\right] = 1$$

$$240\left[\frac{20}{x^2+20x}\right] = 1$$

$$4800 = x^2+20x$$

$$x^2+20x-4800 = 0$$

$$(x+80)(x-60) = 0$$

$$x = -80, 60$$

$\therefore$ (-80 speed cannot be negative)

$$x = 60$$

Average speed of passenger train be 60 km/hr.

Average speed of Express train be 80 km/hr.

Example 3.40

Determine the nature of roots for the following quadratic equation.

(i) $x^2 - x - 20 = 0$ (ii) $9x^2 - 24x + 16 = 0$ (iii) $2x^2 - 2x + 9 = 0$

Sol (i) $x^2 - x - 20 = 0$

$a = 1,\ b = -1,\ c = -20$

$\Delta = b^2 - 4ac$

$= (-1)^2 - 4(1)(-20) = 1 + 80 = 81$

$\Delta > 0$

$\therefore$ The equation will have real and unequal roots.

(ii) $9x^2 - 24x + 16 = 0$

$a = 9,\ b = -24,\ c = 16$

$\Delta = b^2 - 4ac$

$= (-24)^2 - 4(9)(16)$

$\Delta = 576 - 576 = 0$

$\therefore \Delta = 0$, the equation have real and equal roots.

(iii) $2x^2 - 2x + 9 = 0$

$a = 2,\ b = -2,\ c = 9$

$\Delta = b^2 - 4ac$

$= (-2)^2 - 4(2)(9)$

$= 4 - 72$

$\Delta = -68$

$\Delta < 0$

$\therefore$ The equation have no real roots.

Example 3.44

(i) Find the values of 'k' for which the quadratic equation $kx^2-(8k+4)+81=0$ has real and equal roots.

(ii) Find the values of 'k' such that quadratic equation $(k+9)x^2+(k+1)x+1=0$ has no real roots?

Sol (i) $kx^2-(8k+4)x+81=0$

The equation has real and equal roots

$\Delta=0$

$b^2-4ac=0$

$a=k,\ b=-(8k+4),\ c=81$

$(-[8k+4])^2-4(k)(81)=0$

$64k^2+64k+16-324k=0$

$64k^2-260k+16=0$

$\div$ by 4

$16k^2-65k+4=0$

$(16k-1)(k-4)=0$

$k=1/16,\ 4$

(ii) $(k+9)x^2+(k+1)x+1=0$

The equation has no real roots $\Delta<0$

$b^2-4ac<0$

$a=k+9,\ b=k+1,\ c=1$

$(k+1)^2-4(k+9)(1)<0$

$k^2+2k+1-4k-36<0$

$k^2-2k-35<0$

$(k+5)(k-7)<0$

$\therefore -5<k<7$. (If $\alpha<\beta$ and if $(x-\alpha)(x-\beta)<0$ then $\alpha<x<\beta$)

Example 3.42

Prove that the equation $x^2(p^2+q^2)+2x(pr+qs)+r^2+s^2=0$ has no real roots. If $ps=qr$, then show that the roots are real and equal.

Sol

The given quadratic equation

$$x^2(p^2+q^2)+2x(pr+qs)+r^2+s^2=0$$

$$a=p^2+q^2,\ b=(pr+qs)2,\ c=r^2+s^2$$

$$\Delta=b^2-4ac=\left[(2(pr+qs))\right]^2-4(p^2+q^2)(r^2+s^2)$$

$$=4(pr+qs)^2-4(p^2+q^2)(r^2+s^2)$$

$$=4(p^2r^2+q^2s^2+2prqs)-4(p^2r^2+p^2s^2+q^2r^2+q^2s^2)$$

$$=4\left[p^2r^2+q^2s^2+2prqs-p^2r^2-p^2s^2-q^2r^2-q^2s^2\right]$$

$$=4\left[-p^2s^2+2prqs-q^2r^2\right]$$

$$=-4\left[p^2s^2-2prqs+q^2r^2\right]$$

$$=-4\left[(ps-qr)^2\right]<0 \longrightarrow (1)$$

$\Delta=b^2-4ac<0$, the roots are not real

If $ps=qr$ then using (1)

$$\Delta=-4\left[ps-qr\right]^2$$

$$=-4\left[qr-qr\right]^2$$

$$\Delta=0$$

If $ps=qr$ and the roots will be real and equal.

Example 3.43

If the difference between the roots of the equation $x^2 - 13x + k = 0$ is 17 find k.

Sol

$$x^2 - 13k + k = 0$$

$$a = 1,\ b = -13,\ c = k$$

Let α, β be the roots of equation

$$\alpha + \beta = \frac{-b}{a} = \frac{13}{1} = 13 \longrightarrow (1)$$

$$\alpha - \beta = 17 \longrightarrow (2)$$

(1) + (2)

$$2\alpha = 13 + 17$$

$$2\alpha = 30$$

$$\boxed{\alpha = 15}$$

$\alpha = 15$ in (1)

$$15 + \beta = 13$$

$$\beta = 13 - 15 = -2$$

$$\alpha\beta = \frac{c}{a} = \frac{k}{1} = k \Rightarrow 15 \times (-2) = k \ ;\ k = -30$$

Example 3.44

If α and β are the roots of $x^2 + 7x + 10 = 0$ find the value of (i) $(\alpha - \beta)$ (ii) $\alpha^2 + \beta^2$ (iii) $\alpha^3 - \beta^3$ (iv) $\alpha^4 + \beta^4$ (v) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (vi) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

Sol

$$x^2 + 7x + 10 = 0$$

$$a = 1,\ b = 7,\ c = 10$$

If α and β are roots of the equation

$$\alpha + \beta = \frac{-b}{a} = \frac{-7}{1} = -7 \ ;\ \alpha\beta = \frac{c}{a} = \frac{10}{1} = 10$$

(i) $\alpha-\beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{(-7)^2 - 4\times 10} = \sqrt{49-40} = \sqrt{9}$

$\alpha-\beta = 3$

(ii) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (-7)^2 - 2\times 10 = 49-20 = 29$

(iii) $\alpha^8 - \beta^8 = (\alpha-\beta)^3 + 3\alpha\beta(\alpha-\beta)$

$= (+3)^3 + 3(10)(3) = 27+90 = 117$

(iv) $\alpha^4+\beta^4 = (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2 = (29)^2 - 2(10)^2$

$= 841 - 200$

$= 641$

(v) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{49-20}{10} = \frac{29}{10}$

(vi) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$

$= \frac{-343 - 3(10\times(-7))}{10}$

$= \frac{-343+210}{10} = \frac{-133}{10}$

Example 3.45

If α, β are the roots of the equation $3x^2+7x-2=0$ find the values of (i) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (ii) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

Sol

$3x^2+7x-2=0$

$a=3,\ b=+7,\ c=-2$

α, β are the roots of the equation

(i) $\alpha+\beta = \frac{-b}{a} = \frac{-7}{3}$; $\alpha\beta = \frac{c}{a} = \frac{-2}{3}$

(i) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{2\beta\alpha} = \frac{\left(\frac{-7}{3}\right)^2-2\left(\frac{-2}{3}\right)}{\frac{-2}{3}}$

$$= \frac{\frac{49}{9}+\frac{4}{3}}{\frac{-2}{3}}$$

$$= \frac{\frac{49+12}{9}}{\frac{-2}{3}} = \frac{61}{9}\times\frac{-3}{2}$$

$$= \frac{-61}{6}$$

(ii) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}$

$$= \frac{\left(\frac{-7}{3}\right)^3-3\left(\frac{-2}{3}\right)\left(\frac{-7}{3}\right)}{\frac{-2}{3}}$$

$$= \frac{\frac{-343}{27} - \frac{14}{3}}{\frac{-2}{3}} = \frac{\frac{-343-126}{27}}{\frac{-2}{3}} = \frac{-469}{27} \times \frac{-3}{2}$$

$$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{469}{18}$$

Example 3.46

If α, β are the roots of the equation $2x^2 - x - 1 = 0$, then form the equation whose roots are (i) $\frac{1}{\alpha}, \frac{1}{\beta}$ (ii) $\alpha^2\beta, \beta^2\alpha$ (iii) $2\alpha+\beta, 2\beta+\alpha$

Sol

$$2x^2 - x - 1 = 0$$

$$a = 2, \; b = -1, \; c = -1$$

$$\alpha + \beta = \frac{-b}{a} = \frac{1}{2} \; ; \quad \alpha\beta = \frac{c}{a} = \frac{-1}{2}$$

(i) $\frac{1}{\alpha}, \frac{1}{\beta}$

Sum of the roots $= \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \frac{\frac{1}{2}}{-1/2} = \frac{1}{2} \times \frac{-2}{1} = -1$

Product of the root $= \frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{\frac{-1}{2}} = -2$

The required equation is $x^2 -$ (Sum of the roots) $x +$ (product of the roots) $= 0$

$$x^2 - (-1)x + (-2) = 0$$

$$x^2 + x - 2 = 0$$

(ii) Given roots $\alpha^2\beta, \beta^2\alpha$

Sum of the roots $= \alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha+\beta) = \frac{-1}{2}\left(\frac{1}{2}\right) = \frac{-1}{4}$

Product of the roots $= \alpha^2\beta \times \beta^2\alpha = \alpha^3\beta^3 = (\alpha\beta)^3 = \left(\frac{-1}{2}\right)^3 = \frac{-1}{8}$

The require equation is x^2 - (sum of the root) x + product of the roots $= 0$ (102)

$$x^2 - \left(-\frac{1}{4}\right)x - \frac{1}{8} = 0$$

$$\frac{8x^2 + 2x - 1}{8} = 0$$

$$8x^2 + 2x - 1 = 0$$

(iii) $2\alpha + \beta$, $2\beta + \alpha$

sum of the roots $2\alpha + \beta + 2\beta + \alpha = 3\beta + 3\alpha = 3(\alpha + \beta)$

$$= 3\left(\frac{1}{2}\right) = \frac{3}{2}$$

Product of the roots $= (2\alpha + \beta)(2\beta + \alpha)$

$$= 4\alpha\beta + 2\alpha^2 + 2\beta^2 + \alpha\beta$$

$$= 5\alpha\beta + 2(\alpha^2 + \beta^2)$$

$$= 5\alpha\beta + 2\left[(\alpha + \beta)^2 - 2\alpha\beta\right]$$

$$= 5(-1/2) + 2\left[\left(\frac{1}{2}\right)^2 - 2\left(\frac{-1}{2}\right)\right]$$

$$= -\frac{5}{2} + 2\left[\frac{1}{4} + 1\right]$$

$$= -\frac{5}{2} + 2\left[\frac{1+4}{4}\right]$$

$$= -\frac{5}{2} + \frac{5}{2} = 0$$

x^2 - (sum of the root) x + product of the roots $= 0$

$$x^2 - \frac{3}{2}x + 0 = 0$$

$$\frac{2x^2 - 3x}{2} = 0$$

$$2x^2 - 3x = 0$$

Example 3.56

Consider the following Information regarding the number of men and women workers in three factories I, II, III

Factory	Men	women
I	23	18
II	47	36
III	15	16

Represent the above information in the form of a matrix. What does the entry in the second row and first column represent ?

Sol

The information is represented in the form of a 3×2 matrix as follows :

$$A = \begin{pmatrix} 23 & 18 \\ 47 & 36 \\ 15 & 16 \end{pmatrix}$$

The entry in the second row and first column represent that there are 47 men workers.

Example 3.57

If a matrix has 16 elements, what are the possible orders it can have ?

Sol

A matrix of an order m×n, has mn elements.

To find all possible order of a matrix with 16 elements.

Find ordered pairs of natural number whose product is 16.

Such ordered pairs (1, 16), (16, 1), (4, 4), (2, 8), (8, 2)

Hence possible orders are 1×16, 16×1, 4×4, 2×8, 8×2

Example 3.58

Construct a 3x3 matrix whose elements are $a_{ij} = i^2 j^2$

Sol

The general 3x3 matrix $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{34} \end{pmatrix}$

$$a_{ij} = i^2 j^2$$

$a_{11} = 1^2 \times 1^2 = 1$; $a_{21} = 2^2 \times 1^2 = 4$; $a_{31} = 3^2 \times 1^2 = 9$

$a_{12} = 1^2 \times 2^2 = 4$; $a_{22} = 2^2 \times 2^2 = 16$; $a_{32} = 3^2 \times 2^2 = 36$

$a_{13} = 1^2 \times 3^2 = 9$; $a_{23} = 2^2 \times 3^2 = 36$; $a_{33} = 3^2 \times 3^2 = 81$

Hence the required matrix $A = \begin{bmatrix} 1 & 4 & 9 \\ 4 & 16 & 36 \\ 9 & 36 & 81 \end{bmatrix}$

Example 3.59

Find the value of a, b, c, d from the equation

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix}$$

Sol

The given matrix are equal. All corresponding elements are equal.

$a - b = 1 \longrightarrow (1)$

$2a + c = 5 \longrightarrow (2)$

$2a - b = 0 \longrightarrow (3)$

$3c + d = 2 \longrightarrow (4)$

(3) $\Rightarrow$ $2a - b = 0$

$b = 2a \longrightarrow (5)$

(5) in (1)

$a - 2a = 1$

$-a = 1$

$\boxed{a = -1}$

Put $a = -1$ in (5)

$$2(-1) = b$$

$$\boxed{b = -2}$$

Put $a = -1$ in (2)

$$2(-1) + c = 5$$

$$-2 + c = 5$$

$$c = 5 + 2$$

$$\boxed{c = 7}$$

Put $c = 7$ in (4)

$$3(7) + d = 2$$

$$21 + d = 2$$

$$d = 2 - 21$$

$$\boxed{d = -19}$$

$\therefore a = -1, b = -2, c = 7, d = -19$

Example 3.60

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{bmatrix}$, find $A + B$

Sol

$$A + B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+7 & 3+0 \\ 4+1 & 5+3 & 6+1 \\ 7+2 & 8+4 & 9+0 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 2 & 9 & 3 \\ 5 & 8 & 7 \\ 9 & 12 & 9 \end{bmatrix}$$

Example 3.61

Two examinations were conducted for three groups of students namely group 1, group 2, group 3 and their data on average of marks for the subjects Tamil, English, Science, Mathematics are given below in the form of matrices A and B. Find the total mark of both the examination for all the three group.

$$A = \begin{array}{c} \\ \text{Group 1} \\ \text{Group 2} \\ \text{Group 3} \end{array} \begin{array}{c} \begin{array}{cccc} \text{Tamil} & \text{English} & \text{Science} & \text{Mathematics} \end{array} \\ \begin{bmatrix} 22 & 15 & 14 & 23 \\ 50 & 62 & 21 & 30 \\ 53 & 80 & 32 & 40 \end{bmatrix} \end{array}$$

$$B = \begin{array}{c} \\ \text{Group 1} \\ \text{Group 2} \\ \text{Group 3} \end{array} \begin{array}{c} \begin{array}{cccc} \text{Tamil} & \text{English} & \text{Science} & \text{Mathematics} \end{array} \\ \begin{bmatrix} 20 & 38 & 15 & 40 \\ 18 & 12 & 17 & 80 \\ 81 & 47 & 52 & 18 \end{bmatrix} \end{array}$$

Sol

The total mark in both the examination for all the three groups is the sum of the given matrices.

$$A+B = \begin{bmatrix} 22 & 15 & 14 & 23 \\ 50 & 62 & 21 & 30 \\ 53 & 80 & 32 & 40 \end{bmatrix} + \begin{bmatrix} 20 & 38 & 15 & 40 \\ 18 & 12 & 17 & 80 \\ 81 & 47 & 52 & 18 \end{bmatrix}$$

$$= \begin{bmatrix} 22+20 & 15+38 & 14+15 & 23+40 \\ 50+18 & 62+12 & 21+17 & 30+80 \\ 53+81 & 80+47 & 32+52 & 40+18 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 42 & 53 & 29 & 63 \\ 68 & 74 & 38 & 110 \\ 134 & 127 & 84 & 58 \end{bmatrix}$$

Example 3.62.

If $A = \begin{pmatrix} 1 & 3 & -2 \\ 5 & -4 & 6 \\ -3 & 2 & 9 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 8 \\ 3 & 4 \\ 9 & 6 \end{pmatrix}$, find $A+B$.

Sol

It is not possible to add A and B because they have different orders.

Example 3.63

If $A = \begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$ Find $2A+B$.

Sol

Since A and B have same order 3×3, $2A+B$ is defined.

$$2A+B = 2\begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 16 & 12 \\ 2 & 6 & 18 \\ -8 & 6 & -2 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 14+4 & 16+11 & 12-3 \\ 2-1 & 6+2 & 18+4 \\ -8+7 & 6+5 & -2+0 \end{pmatrix}$$

$$2A+B = \begin{pmatrix} 18 & 27 & 9 \\ 1 & 8 & 22 \\ -1 & 11 & -2 \end{pmatrix}$$

Example 3.64

If $A = \begin{pmatrix} 5 & 4 & -2 \\ 1/2 & 3/4 & \sqrt{2} \\ 1 & 9 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -7 & 4 & 3 \\ 1/4 & 7/2 & 3 \\ 5 & -6 & 9 \end{pmatrix}$, find $4A-3B$.

Sol

Since A, B are of the same order 3×3, subtraction of 4A and 3B is defined

$$4A - 3B = 4\begin{bmatrix} 5 & 4 & -2 \\ 1/2 & 3/4 & \sqrt{2} \\ 1 & 9 & 4 \end{bmatrix} - 3\begin{bmatrix} -7 & 4 & -3 \\ 1/4 & 7/2 & 3 \\ 5 & -6 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 16 & -8 \\ 2 & 3 & 4\sqrt{2} \\ 4 & 36 & 16 \end{bmatrix} - \begin{bmatrix} -21 & 12 & -9 \\ 3/4 & 21/2 & 9 \\ 15 & -18 & 27 \end{bmatrix}$$

$$= \begin{bmatrix} 20+21 & 16-12 & -8+9 \\ 2-3/4 & 3-21/2 & 4\sqrt{2}-9 \\ 4-15 & 36+18 & 16-27 \end{bmatrix}$$

$$4A-3B = \begin{bmatrix} 41 & 4 & 1 \\ 5/4 & -15/2 & 4\sqrt{2}-9 \\ -11 & 54 & -11 \end{bmatrix}$$

Example 3.65

Find the value of a, b, c, d from the following matrix equation.

$$\begin{bmatrix} d & 8 \\ 3b & a \end{bmatrix} + \begin{bmatrix} 3 & a \\ -2 & -4 \end{bmatrix} = \begin{bmatrix} 2 & 2a \\ b & 4c \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -5 & 0 \end{bmatrix}$$

Sol

Add two matrices on both side

$$\begin{bmatrix} d+3 & 8+a \\ 3b-2 & a-4 \end{bmatrix} = \begin{bmatrix} 2 & 2a+1 \\ b-5 & 4c \end{bmatrix}$$

Equating the corresponding elements.

$d+3=2$

$d = 2-3 \Rightarrow d=-1$

$$8+a = 2a+1$$

$$8-1 = 2a-a$$

$$\boxed{a=7}$$

$$3b-2 = b-5$$

$$3b-b = -5+2$$

$$2b = -3$$

$$\boxed{b = -3/2}$$

Substitute $a=7$ in $a-4 = 4c$

$$7-4 = 4c$$

$$3 = 4c$$

$$c = 3/4$$

$$\therefore a=7,\ b=-3/2,\ c=3/4,\ d=-1$$

Example 3.66

If $A = \begin{bmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{bmatrix}$

compute the following: (i) $3A+2B-C$ (ii) $1/2A - 3/2B$

Sol

(i) $$3A+2B-C = 3\begin{bmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{bmatrix} + 2\begin{bmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 24 & 9 \\ 9 & 15 & 0 \\ 24 & 21 & 18 \end{bmatrix} + \begin{bmatrix} 16 & -12 & -8 \\ 4 & 22 & -6 \\ 0 & 2 & 10 \end{bmatrix} - \begin{bmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3+16-5 & 24-12-3 & 9-8+0 \\ 9+4+1 & 15+22+7 & 0+6-2 \\ 24+0-1 & 21+2-4 & 18+10-3 \end{bmatrix}$$

$$3A+2B-C = \begin{bmatrix} 14 & 9 & 1 \\ 14 & 44 & -8 \\ 23 & 19 & 25 \end{bmatrix}$$

(ii) $\frac{1}{2}A - \frac{3}{2}B = \frac{1}{2}(A-3B)$

$$= \frac{1}{2}\left[\begin{bmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{bmatrix} - 3\begin{bmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{bmatrix}\right]$$

$$= \frac{1}{2}\left[\begin{bmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{bmatrix} - \begin{bmatrix} 24 & -18 & -12 \\ 6 & 33 & -9 \\ 0 & 3 & 15 \end{bmatrix}\right]$$

$$= \frac{1}{2}\left[\begin{bmatrix} 1-24 & 8+18 & 3+12 \\ 3-6 & 5-33 & 0+9 \\ 8-0 & 7-3 & 6-15 \end{bmatrix}\right]$$

$$= \frac{1}{2}\begin{bmatrix} -23 & 26 & 15 \\ -3 & -28 & 9 \\ 8 & 4 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} -23/2 & 13 & 15/2 \\ -3/2 & -14 & 9/2 \\ 4 & 2 & -9/2 \end{bmatrix}$$

Example 3.67

If $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{bmatrix}$, find AB

Sol

A is 2×3 matrix, B is 3×3 matrix, hence AB is defined.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{bmatrix}_{2\times3} \qquad B = \begin{bmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{bmatrix}_{3\times3}$$

$$AB = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{pmatrix} \times \begin{pmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} 8+4+0 & 3+8+0 & 1+2+0 \\ 24+2+25 & 9+4+15 & 3+1+5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 12 & 11 & 3 \\ 51 & 28 & 9 \end{bmatrix}$$

Example 3.68

If $A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$ find AB and BA.

Verify $AB = BA$?

Sol

A is a 2×2 matrix, B is a 2×2 matrix. Hence AB is defined. It will be the order of 2×2.

$$AB = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \times \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} = \begin{bmatrix} 4+1 & 0+3 \\ 2+3 & 0+9 \end{bmatrix}$$

$$AB = \begin{bmatrix} 5 & 3 \\ 5 & 9 \end{bmatrix}$$

$$BA = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} = \begin{bmatrix} 4+0 & 2+0 \\ 2+3 & 1+9 \end{bmatrix}$$

$$BA = \begin{bmatrix} 4 & 2 \\ 5 & 10 \end{bmatrix}$$

$\therefore AB \neq BA$

Example 3.69.

If $A = \begin{bmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{bmatrix}$

Show that A and B satisfy commutative property with respect to matrix multiplication.

Sol: to show that $AB = BA$

LHS: AB

$$AB = \begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 4 + (2\times 2) & 4\sqrt{2} - 4\sqrt{2} \\ 2\sqrt{2} - 2\sqrt{2} & 4 + 4 \end{pmatrix}$$

$$AB = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$$

RHS: BA

$$BA = \begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{pmatrix} \times \begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 4+4 & -4\sqrt{2} + 4\sqrt{2} \\ -2\sqrt{2}+2\sqrt{2} & 4+4 \end{pmatrix}$$

$$BA = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$$

$\therefore$ LHS = RHS

$AB = BA$

Example 3.70

solve $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$

Sol

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}_{2\times 2} \times \begin{pmatrix} x \\ y \end{pmatrix}_{2\times 1} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$ (113)

$$\begin{pmatrix} 2x+y \\ x+2y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

$2x+y = 4 \longrightarrow (1)$

$x+2y = 5 \longrightarrow (2)$

Equation (1) × 2, solve (1) & (2)

(1) ⇒ $4x + 2y = 8$

(2) ⇒ $x + 2y = 5$

$\ominus \quad \ominus \quad \ominus$

$3x = 3$

$x = 3/3$

$\boxed{x = 1}$

$x = 1$ in (2)

$1 + 2y = 5$

$2y = 5 - 1$

$2y = 4$

$y = 4/2$

$\boxed{y = 2}$

$\therefore x = 1, y = 2.$

Example 3.71

If $A = \begin{pmatrix} 1 & -1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$

show that $(AB)C = A(BC)$

Sol

LHS : $(AB)C$

$$AB = \begin{pmatrix} 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1-2+2 & -1-1+6 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 4 \end{pmatrix}$$

$$(AB)C = \begin{pmatrix} 1 & 4 \end{pmatrix} \times \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+8 & 2+4 \end{pmatrix}$$

$$(AB)C = \begin{pmatrix} 9 & -2 \end{pmatrix} \longrightarrow (1)$$

RHS = $A(BC)$

$$BC = \begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix} \times \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-2 & 2+1 \\ 2+2 & 4-1 \\ 1+6 & 2-3 \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ 4 & 3 \\ 7 & -1 \end{pmatrix}$$

$$A(BC) = \begin{pmatrix} 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} -1 & 3 \\ 4 & 3 \\ 7 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -1-4+14 & 3-3-2 \end{pmatrix}$$

$$A(BC) = \begin{pmatrix} 9 & -2 \end{pmatrix} \longrightarrow (2)$$

From (1) & (2)

$(AB)C = A(BC)$, Hence proved.

Example 3.72

If $A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix}$, $C = \begin{pmatrix} -1 & 6 \\ 3 & 2 \end{pmatrix}$

verify that $A(B+C) = AB + AC$.

Sol

LHS: $A(B+C)$

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$$B+C = \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix} + \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} -6 & 8 \\ -1 & 4 \end{pmatrix}$$

$$A(B+C) = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \times \begin{pmatrix} -6 & 8 \\ -1 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} -6-1 & 8+4 \\ 6-3 & -8+12 \end{pmatrix}$$

$$A(B+C) = \begin{pmatrix} -7 & 12 \\ 3 & 4 \end{pmatrix} \longrightarrow (1)$$

RHS: $AB + AC$

$$AB = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \times \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix} = \begin{pmatrix} 1-4 & 2+2 \\ -1-12 & -2+6 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 4 \\ -13 & 4 \end{pmatrix}$$

$$AC = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \times \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} -7+3 & 6+2 \\ 7+9 & -6+6 \end{pmatrix}$$

$$= \begin{pmatrix} -4 & 8 \\ 16 & 0 \end{pmatrix}$$

$$AB + AC = \begin{pmatrix} -3 & 4 \\ -13 & 4 \end{pmatrix} + \begin{pmatrix} -4 & 8 \\ 16 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -3-4 & 4+8 \\ -13+16 & 4+0 \end{pmatrix}$$

$$AB + AC = \begin{pmatrix} -7 & 12 \\ +3 & 4 \end{pmatrix} \longrightarrow (2)$$

From (1) & (2)

$$A(B+C) = AB + AC$$

Hence proved.

Example 3.13

If $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}$ show that

$(AB)^T = B^T A^T$

Sol

LHS : $(AB)^T$

$$AB = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix} \times \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 2-2+0 & -1+8+2 \\ 4+1+0 & -2-4+2 \end{pmatrix}$$

$$AB = \begin{pmatrix} 0 & 9 \\ 5 & -4 \end{pmatrix}$$

$$(AB)^T = \begin{pmatrix} 0 & 5 \\ 9 & -4 \end{pmatrix} \longrightarrow (1)$$

RHS : $B^T A^T$

$$B^T = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{pmatrix} ; \quad A^T = \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{pmatrix} \times \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2-2+0 & 4+1+0 \\ -1+8+2 & -2-4+2 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 0 & 5 \\ 9 & -4 \end{pmatrix} \longrightarrow (2)$$

From (1) & (2)

$(AB)^T = B^T A^T$

Hence proved.

Xth HAND WRITTEN MATERIALS

CHAPTER – 4 (Examples)

GEOMETRY

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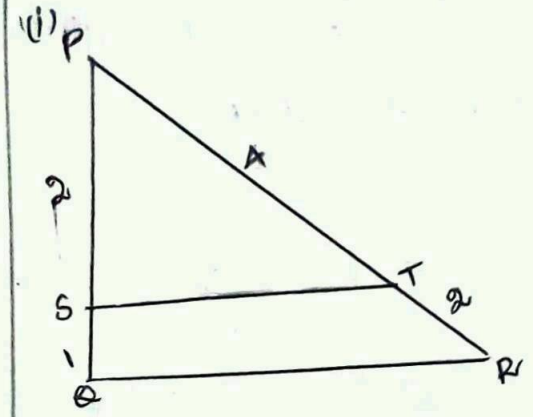
CHAPTER - 4

GEOMETRY

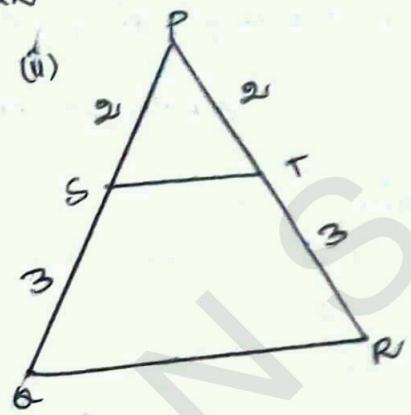
Example 4.1

Show that $\Delta PST \sim \Delta PQR$

(i)



(ii)



Sol

(i) In ΔPST and ΔPQR,

$$\frac{PS}{PQ} = \frac{2}{2+1} = \frac{2}{3}, \quad \frac{PT}{PR} = \frac{4}{4+2} = \frac{2}{3}$$

Thus, $\frac{PS}{PQ} = \frac{PT}{PR}$ and $\angle P$ is common

$\therefore$ by SAS similarity,

$\Delta PST \sim \Delta PQR$.

(ii) In ΔPST and ΔPQR

$$\frac{PS}{SQ} = \frac{2}{2+3} = \frac{2}{5}, \quad \frac{PT}{PR} = \frac{2}{2+3} = \frac{2}{5}$$

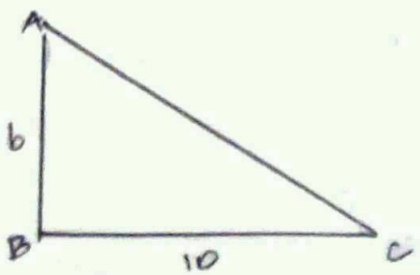
Thus, $\frac{PS}{PQ} = \frac{PT}{PR}$ and $\angle P$ is common

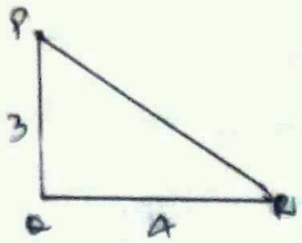
$\therefore$ By SAS similarity,

$\Delta PST \sim \Delta PQR$

Example 4.2

Is $\Delta ABC \sim \Delta PQR$?





Sol In ΔABC and ΔPQR,

$$\frac{PQ}{AB} = \frac{3}{6} = \frac{1}{2} \; ; \; \frac{QR}{BC} = \frac{4}{10} = \frac{2}{5}$$

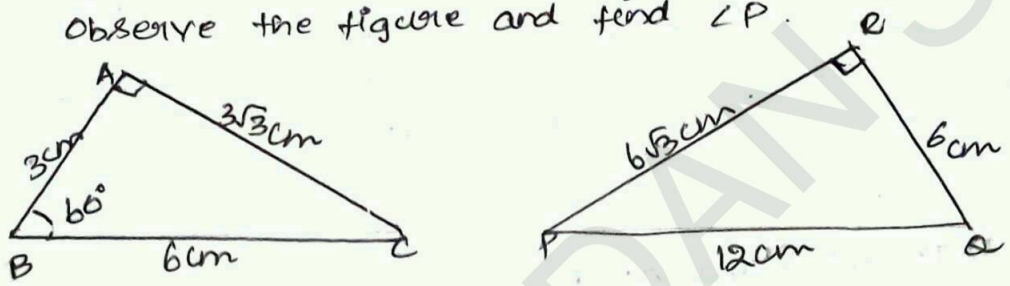
since $\frac{1}{2} \neq \frac{2}{5}$, $\frac{PQ}{AB} \neq \frac{QR}{BC}$

The corresponding sides are not proportional.

$\therefore$ ΔABC is not similar to ΔPQR

Example 4.3

Observe the figure and find $\angle P$.



Sol In ΔBAC and ΔPRQ,

$$\frac{AB}{RQ} = \frac{3}{6} = \frac{1}{2} \; ; \; \frac{BC}{QP} = \frac{6}{12} = \frac{1}{2} \; ; \; \frac{CA}{PR} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$$

$$\therefore \frac{AB}{RQ} = \frac{BC}{QP} = \frac{CA}{PR}$$

By SSS similarity, we have $\Delta BAC \sim \Delta QRP$

$\angle P = \angle C$ (since the corresponding parts of similar triangle)

$\angle P = \angle C = 180° - (\angle A + \angle B)$

$\angle P = 180° - (90° + 60°)$

$\angle P = 180° - 150°$

$\angle P = 30°$

Example 4.4

A boy of height 90cm is walking away from the base of a lamp post at a speed of 1.2 m/sec. If the lamppost is 3.6m above the ground, find the length of his shadow case after 4 seconds.

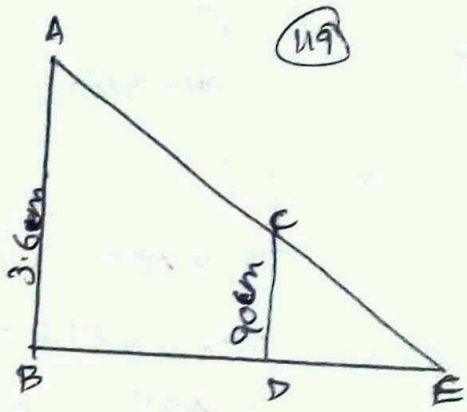
Sol Given, speed $= 1.2$ m/s

time $= 4$ seconds.

distance = Speed × time

$= 1.2 \times 4$

$= 4.8\,m$



Let x be the length of the shadow after 4 seconds.

Since, $\Delta ABE \sim \Delta CDE$, $\frac{BE}{DE} = \frac{AB}{CD}$

$$\frac{4.8 + x}{x} = \frac{3.6}{0.9} \quad [\because 90\,cm = 0.9\,m]$$

$$\frac{4.8 + x}{x} = 4$$

$$4.8 + x = 4x$$

$$4.8 = 4x - x$$

$$4.8 = 3x$$

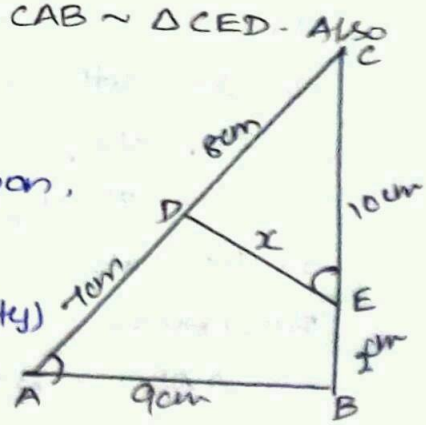
$$x = \frac{4.8}{3}$$

$$\boxed{x = 1.6}$$

$\therefore$ The length of his shadow $DE = 1.6\,m$.

Example 4.5

In Fig, $\angle A = \angle CED$ prove that $\Delta CAB \sim \Delta CED$. Also find the value of x



Sol In ΔCAB and ΔCED, $\angle C$ is common,

$\angle A = \angle CED$

$\therefore \Delta CAB \sim \Delta CED$ (By AA similarity)

$$\frac{CA}{CE} = \frac{AB}{DE} = \frac{CB}{CD}$$

$$\frac{AB}{DE} = \frac{CB}{CD}$$

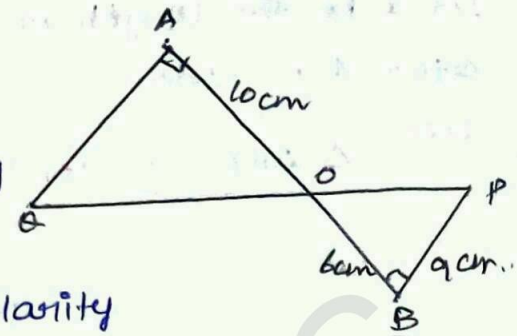
$$\frac{9}{x} = \frac{10+2}{8} \Rightarrow \frac{9}{x} = \frac{12}{8} \Rightarrow \frac{9}{x} = \frac{3}{2} \Rightarrow 18 = 3x$$

$x = 18/3 = 6\,cm$.

Example 4.6

In the figure, QA and PB are perpendicular to AB. If AO = 10cm, BO = 6cm and PB = 9cm, find AQ.

Sol



In ΔAOQ and ΔBOP,

$\angle OAQ = \angle OBP = 90^\circ$

$\angle AOQ = \angle BOP$ (vertically opposite angle)

$\therefore$ By AA criterion of similarity

$\Delta AOQ \sim \Delta BOP$

$$\frac{AO}{BO} = \frac{OQ}{OP} = \frac{AQ}{BP}$$

$$\frac{10}{6} = \frac{AQ}{BP}$$

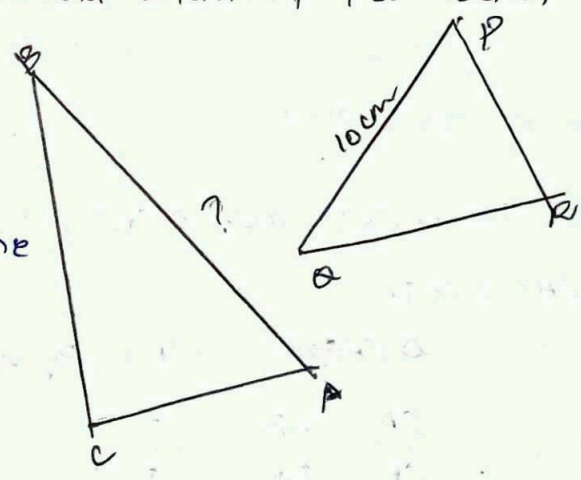
$$\frac{5}{3} = \frac{AQ}{9} \Rightarrow 45 = 3AQ$$

$AQ = 45/3$

$AQ = 15cm$.

Example 4.7

The perimeter of two similar triangles ABC and PQR are respectively 36cm and 24cm. If PQ = 10cm, find AB.



Sol The ratio of the corresponding sides of similar triangles is same as the ratio of their perimeter.

$\Delta ABC \sim \Delta PQR$,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{36}{24}$$

$$\frac{AB}{PQ} = \frac{36}{24} \Rightarrow \frac{AB}{10} = \frac{3}{2} \Rightarrow 2AB = 30$$

$AB = 30/2$

$AB = 15cm$.

Example 4.8

If ΔABC is similar to ΔDEF such that $BC = 3cm$, $EF = 4cm$ and area of $\Delta ABC = 54 cm^2$. Find the area of ΔDEF.

Sol

The ratio of area of two similar triangle is equal to the ratio of the square of any two corresponding sides,

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{BC^2}{EF^2} \Rightarrow \frac{54}{\text{Area}(\Delta DEF)} = \frac{3^2}{4^2}$$

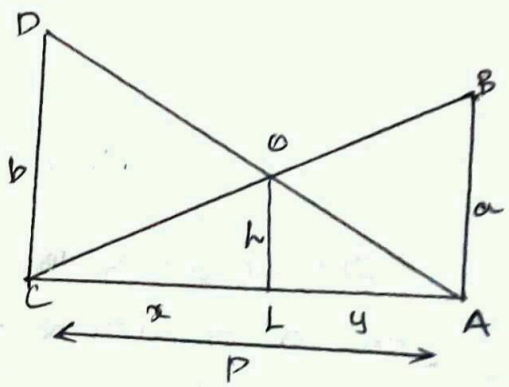
$$\text{Area}(\Delta DEF) = \frac{16 \times 54}{9}$$

$$\text{Area}(\Delta DEF) = 96 cm^2$$

Example 4.9

Two poles of height 'a' meters and 'b' meters are 'p' meter apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ meters.

Sol



Let AB and CD be two poles of height 'a' metres and 'b' metres respectively. such that the poles are 'p' metres apart. that is $AC = P$ metres. suppose the lines AD and BC meet at O, such that $OL = h$ metres.

Let $CL = x$ and $LA = y$

Then $x + y = P$

In ΔABC and ΔLOC,

$\angle CAB = \angle CLO$ (each equal to $90°$)

$\angle C = \angle C$ [C is common]

$\Delta CAB \sim \Delta CLO$ [By AA similarity]

$$\frac{CA}{CL} = \frac{AB}{LO} \Rightarrow \frac{P}{x} = \frac{a}{h}$$

$$x = \frac{Ph}{a} \longrightarrow (1)$$

In ΔALO and ΔACD

$\angle ALO = \angle ACD$ [each equal to 90°]

$\angle A = \angle A$ [A is common]

$\Delta ALO \sim \Delta ACD$ [By AA similarity]

$$\frac{AL}{AC} = \frac{OL}{DC} \Rightarrow \frac{y}{P} = \frac{h}{b}$$

$$y = \frac{Ph}{b} \longrightarrow (2)$$

(1) + (2) $\Rightarrow$ $x + y = \frac{Ph}{a} + \frac{Ph}{b}$ [$\because x+y=P$]

$$P = Ph\left(\frac{1}{a} + \frac{1}{b}\right)$$

$$\frac{P}{P} = h\left(\frac{1}{a} + \frac{1}{b}\right)$$

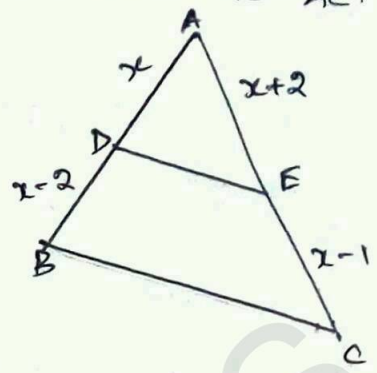
$$1 = h\left(\frac{b+a}{ab}\right)$$

$$\therefore h = \frac{ab}{a+b}$$

The height of the intersection of the lines joining the top of each pole to the foot of the opposite pole is $\frac{ab}{a+b}$ metres.

Example 4.12

In ΔABC, If $DE \parallel BC$, $AD = x$, $DB = x-2$, $AE = x+2$ and $EC = x-1$ then find the length of the sides $AB = AC$.



Sol In ΔABC, $DE \parallel BC$

By Thales theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{x-2} = \frac{x+2}{x-1}$$

$$x(x-1) = (x+2)(x-2)$$

$$x^2 - x = x^2 - 4$$

$$x^2 - x^2 + 4 = x$$

$$\boxed{x = 4}$$

when $x = 4$

$AD = x = 4$, $DB = x-2 = 4-2 = 2$; $AE = x+2 = 4+2 = 6$, $EC = x-1 = 4-1 = 3$.

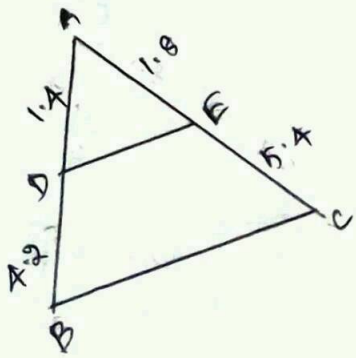
$AB = AD + DB = 4+2 = 6$

$AC = AE + EC = 6+3 = 9$

$\therefore AB = 6cm$, $AC = 9cm$.

Example 4.13

D and E are respectively the points on the sides AB and AC of a ΔABC such that $AB = 5.6cm$, $AD = 1.4cm$, $AC = 4.2cm$, $AE = 1.8cm$. Show that $DE \parallel BC$.



Sol $AB = 5.6cm$, $AD = 1.4cm$, $AC = 4.2cm$, $AE = 1.8cm$.

$BD = AB - AD = 5.6 - 1.4 = 4.2cm$

$EC = AC - AE = 7.2 - 1.8 = 5.4cm$

$$\frac{AD}{DB} = \frac{1.4}{4.2} = \frac{1}{3} \text{ and } \frac{AE}{EC} = \frac{1.8}{5.4} = \frac{1}{3}$$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

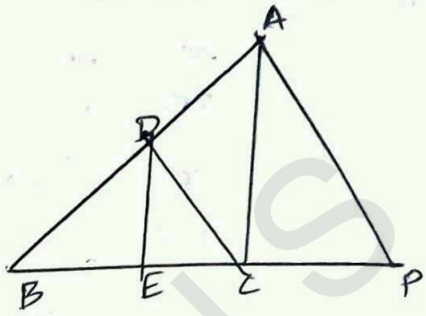
∴ By converse of Basic proportionality theorem, we have DE is parallel to BC.

Hence proved.

Example 4.14

In the fig, DE || AC and DC || AP. prove that

$$\frac{BE}{EC} = \frac{BC}{CP}$$



Sol In ΔBPA, DC || AP. By Basic proportionality theorem.

$$\frac{BC}{CP} = \frac{BD}{DA} \rightarrow (1)$$

In Δ BCA, DE || AC, By Basic proportionality theorem

$$\frac{BE}{EC} = \frac{BD}{DA} \rightarrow (2)$$

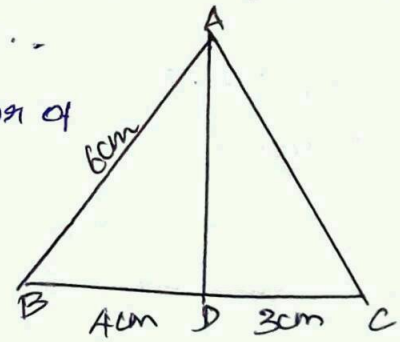
From (1) & (2)

$$\frac{BE}{EC} = \frac{BC}{CP}$$

Hence proved.

Example 4.15

In the fig, AD is the bisector of ∠A. If BD = 4cm, DC = 3cm and AB = 6cm. find AC.



Sol In ΔABC, AD is the bisector of ∠A.

By Angle Bisector theorem,

$$\frac{BD}{DC} = \frac{AB}{AC}$$

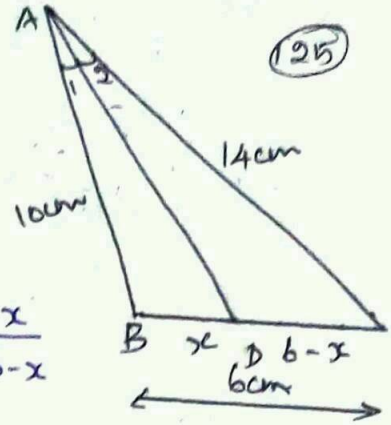
$$\frac{4}{3} = \frac{6}{AC} \Rightarrow 4AC = 18 \Rightarrow AC = \frac{18}{4} = \frac{9}{2} = 4.5cm.$$

Example 4.16

In the fig, AD is the bisector of ∠BAC, If AB = 10cm, AC = 14cm and BC = 6cm. Find BD and DC.

Sol Let BD = x cm then DC = (6 − x) cm

AD is bisector of $\angle A$



By Angle Bisector theorem

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{10}{14} = \frac{x}{6-x} \Rightarrow \frac{5}{7} = \frac{x}{6-x}$$

$$(6-x)5 = 7x$$

$$30 - 5x = 7x$$

$$30 = 7x + 5x$$

$$30 = 12x$$

$$x = 30/12 = 2.5\text{cm}.$$

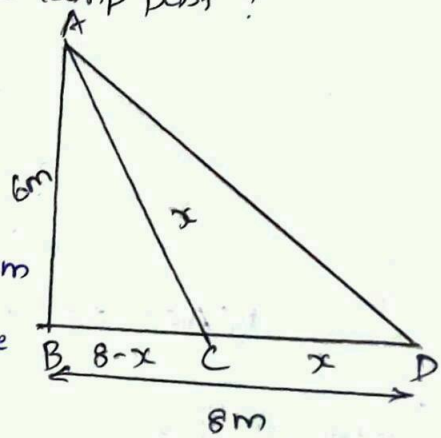
$\therefore$ BD = 2.5cm, DC = 6 − x = 6 − 2.5 = 3.5cm.

Example 4.20.

An insect 8m away initially from the foot of a lamp post which is 6m tall, crawls towards it moving through a distance. If its distance from the top of the lamp post is equal to the distance it has moved, how far is the insect away from the foot of the lamp post?

Sol Distance between the insect and the foot of the lamp post

BD = 8m



The height of the lamp post, AB = 6m

After moving a distance of x m, the insect be at C

Let AC = CD = x, BC = BD − CD = 8 − x

In ΔABC, $\angle B = 90°$

$$AC^2 = AB^2 + BC^2$$

$$x^2 = 6^2 + (8-x)^2$$

$$x^2 = 36 + 64 - 16x + x^2$$

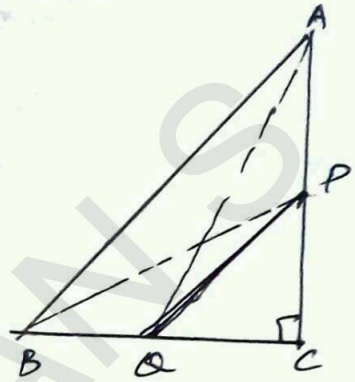
$$16x = 100 \Rightarrow x = 100/16 = 6.25$$

$BC = 8 - x = 8 - 6.25 = 1.75m$

$\therefore$ The insect is 1.75m away from the foot of the lamp post.

Example 4.21

P and Q are the mid-points of the sides CA and AB respectively of ΔABC, right angled at C. Prove that $4(AQ^2 + BP^2) = 5AB^2$



Sol ΔAQC is right triangle at C,

$AQ^2 = AC^2 + QC^2 \longrightarrow (1)$

ΔBPC is right triangle at C

$BP^2 = BC^2 + CP^2 \longrightarrow (2)$

ΔABC is right triangle at C

$AB^2 = AC^2 + BC^2 \longrightarrow (3)$

From (1) & (2)

$AQ^2 + BP^2 = AC^2 + QC^2 + BC^2 + CP^2$

$4(AQ^2 + BP^2) = 4AC^2 + 4QC^2 + 4BC^2 + 4CP^2$

$= 4AC^2 + (2QC)^2 + 4BC^2 + (2CP)^2$

$= 4AC^2 + BC^2 + 4BC^2 + AC^2$

$= 4AC^2 + AC^2 + 4BC^2 + BC^2$ (Since P and Q are midpoint).

$= 5(AC^2 + BC^2)$

$4(AQ^2 + BP^2) = 5AB^2$.

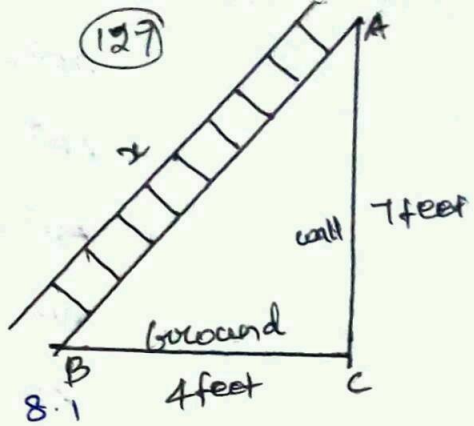
Example 4.22.

What length of ladder is needed to reach a height of 7 ft along the wall when the base of the ladder is 4 ft from the wall? Round off your answer to the next tenth place.

Sol Let x be the length of the ladder.

$BC = 4ft$, $AC = 7ft$.

By pythagoras theorem



$$AB^2 = AC^2 + BC^2$$

$$x^2 = 7^2 + 4^2$$

$$x^2 = 49 + 16$$

$$x^2 = \sqrt{65}$$

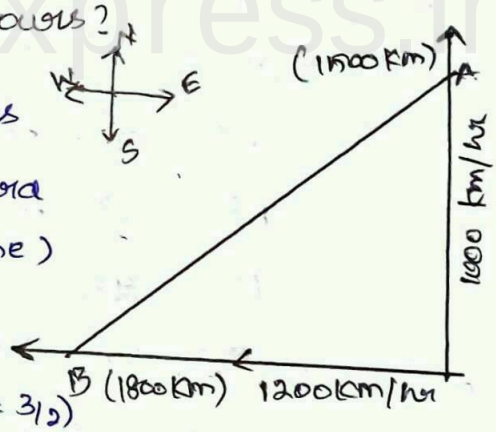
The number $\sqrt{65}$ between 8 and 8.1

$$8^2 = 64 < 65 < 65.61 = (8.1)^2$$

$\therefore$ The length of the ladder is approximately 8.1 ft.

Example 4.23

An Aeroplane after take off from an airport and flies due north at a speed of 1000 Km/hr. At the same time, another aeroplane take off from the same airport ~~will be the two planes after~~ and flies due west at a speed of 1200 Km/hr. How far apart will be the two planes after 1 1/2 hours?



Sol Let the first aeroplane starts from O and goes upto A toward north, (Distance = Speed x time)

$$OA = \left(1000 \times \frac{3}{2}\right) \text{ km}$$

$(\because 1\frac{1}{2} = 3/2)$

$$OA = 1500 \text{ km}$$

Let the second aeroplane starts from O at the same time and goes upto B toward west,

$$OB = \left(1200 \times \frac{3}{2}\right) \text{ km}$$

$$OB = 1800 \text{ km}.$$

The required distance to be found is BA.

In right angle triangle, AOB, $AB^2 = OA^2 + OB^2$

$$AB^2 = (1500)^2 + (1800)^2$$

$$AB^2 = 100^2 (15^2 + 18^2)$$

$$AB^2 = 100^2 (225 + 324)$$

$$AB^2 = 100^2 \times 549$$

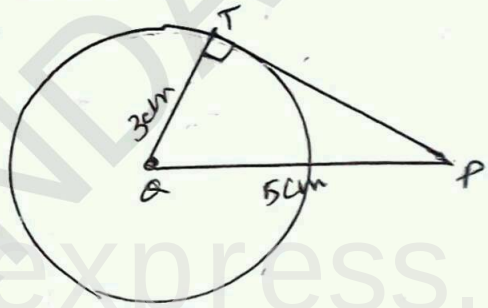
$$AB^2 = 100^2 \times 9 \times 61$$

$$AB = 100 \times 3 \times \sqrt{61}$$

$$AB = 300\sqrt{61} \text{ Kms.}$$

Example 4.24

Find the length of the tangent drawn from a point whose distance from the centre of a circle is 5cm and radius of the circle is 3cm.



Sol

Given: OP = 5cm,

radius r = 3cm

To find the length of tangent PT

In right angled ΔOTP

$$OP^2 = OT^2 + PT^2 \text{ (By Pythagoras theorem)}$$

$$5^2 = 3^2 + PT^2$$

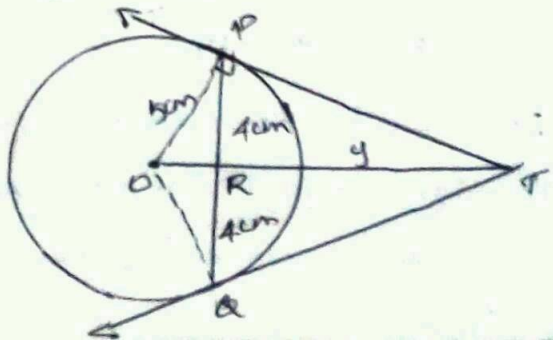
$$PT^2 = 25 - 9$$

$$PT^2 = 16 \Rightarrow PT = 4$$

Length of the tangent PT = 4cm.

Example 4.25

PQ is a chord of length 8cm to a circle of radius 5cm. the tangent at P and Q intersect at a point T. Find the length of the tangent TP.



Sol

Let TR = y, OT is perpendicular bisector of PQ.

PR = QR = 4cm.

OP = 5cm.

In ΔORP, $OP^2 = OR^2 + PR^2$

$$OR^2 = OP^2 - PR^2$$

$$OR^2 = 5^2 - 4^2 = 25 - 16 = 9$$

$$OR = 3cm$$

$$OT = OR + RT = 3 + y \rightarrow (1)$$

In ΔPRT, $TP^2 = TR^2 + PR^2 \rightarrow (2)$

and ΔOPT; $OT^2 = TP^2 + OP^2$

$$OT^2 = (TR^2 + PR^2) + OP^2 \quad (\text{From } (2))$$

$$(3+y)^2 = y^2 + 4^2 + 5^2 \quad (\text{substitute } OT \text{ from } (1))$$

$$9 + 6y + y^2 = y^2 + 16 + 25$$

$$6y = y^2 + 16 + 25 - 9 - y^2$$

$$6y = 41 - 9$$

$$6y = 32$$

$$y = 32/6 = 16/3$$

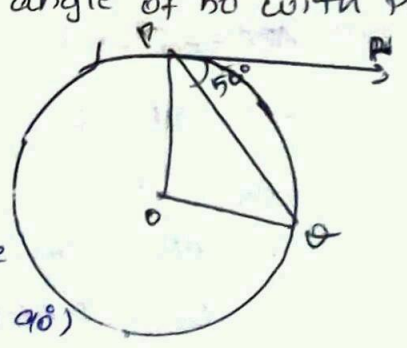
From (2) $TP^2 = TR^2 + PR^2$

$$TP^2 = \left(\frac{16}{3}\right)^2 + 4^2 = \frac{256}{9} + 16$$

$$TP = \frac{256 + 144}{9} = \frac{400}{9} = \frac{20}{3} cm$$

Example 4.26

In fig, O is the centre of a circle. PQ is a chord and tangent PR at P makes an angle of 50° with PQ. Find $\angle POQ$.



Sol

$\angle OPQ = 90° - 50° = 40°$

(angle between the radius and tangent 90°)

$OP = OQ$ (radii of a circle are equal)

$\angle OPQ = \angle OQP = 40^\circ$ (ΔOPQ is isoceles)

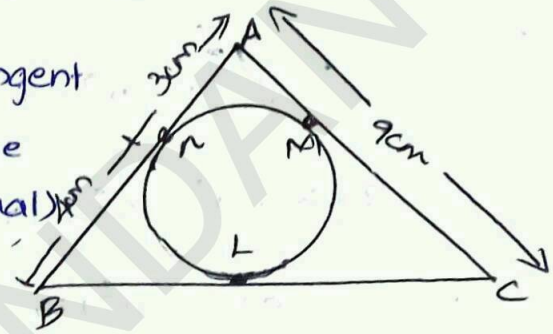
$\angle POQ = 180^\circ - \angle OPQ - \angle OQP$ (130)

$\angle POQ = 180^\circ - 40^\circ - 40^\circ$

$\angle POQ = 100^\circ$

Example 4.27

In fig, ΔABC is circumscribing a circle. Find the length of BC.



Sol

$AN = AM = 3cm$ (Tangent drawn from same exterior point are equal)

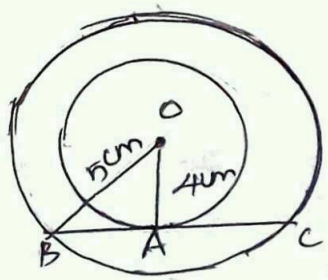
$BN = BL = 4cm$

$CL = CM = AC - AM = 9cm - 3cm = 6cm$.

$BC = BL + CL = 4 + 6 = 10cm$.

Example 4.28

If radii of two concentric circle are 4cm and 5cm then find the length of the chord of one circle which is a tangent to the other circle.



Sol

$OA = 4cm$, $OB = 5cm$

$OA \perp BC$

$OB^2 = OA^2 + AB^2$

$5^2 = 4^2 + AB^2$

$AB^2 = -4^2 + 5^2$

$AB^2 = 25 - 16$

$AB^2 = 9$

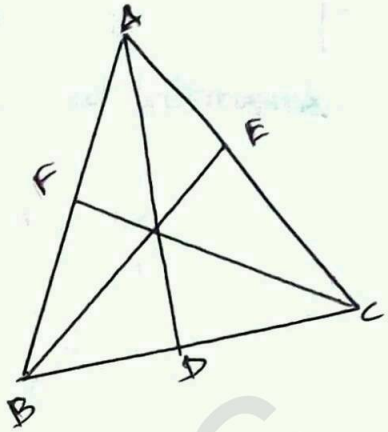
$\therefore AB = 3cm$

$BC = 2AB$ hence $BC = 2 \times 3 = 6cm$.

Example 4.32

Show that in a triangle, the median are concurrent.

Sol



Medians are line segments joining each vertex to the midpoint of the corresponding opposite sides.

Thus median are the cevians where D, E, F are midpoints of BC, CA and AB respectively.

Since D is the midpoint of BC, BD = DC

$$\frac{BD}{DC} = 1 \longrightarrow (1)$$

Since E is the midpoint of AC, ~~AE = AC~~ CE = EA

$$\frac{CE}{EA} = 1 \longrightarrow (2)$$

Since F is the midpoint of AB, AF = FB

$$\frac{AF}{FB} = 1 \longrightarrow (3)$$

multiplying (1), (2), (3)

$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1 \times 1 \times 1 = 1$$

Ceva's theorem is satisfied

∴ The medians are concurrent.

Example 4.33

Suppose AB, AC and BC have length 13, 14 and 15 respectively. If $\frac{AF}{FB} = \frac{2}{5}$ and $\frac{CE}{EA} = \frac{5}{8}$, Find BD and DC.

Sol

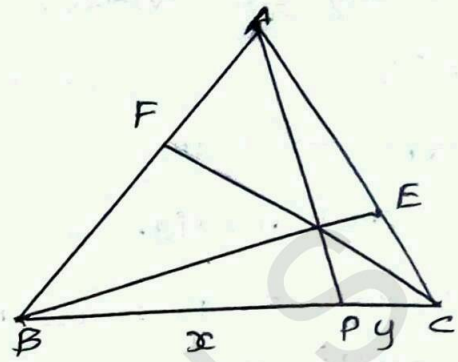
Given that AB = 13, AC = 14, BC = 15

Let BD = x and DC = y

using ceva's theorem,

$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1 \longrightarrow (1)$$

Substitute the value of $\frac{AF}{FB}$ and $\frac{CE}{EA}$ in (1)



$$\frac{BD}{DC} \times \frac{5}{8} \times \frac{2}{5} = 1$$

$$\frac{x}{y} \times \frac{1}{4} = 1$$

$$x = 4y \rightarrow (2)$$

$$BC = BD + DC = 15$$

$$x + y = 15 \longrightarrow (3)$$

From (2) $x = 4y$ in (3)

$$4y + y = 15$$

$$5y = 15$$

$$y = 15/5$$

$$\boxed{y = 3}$$

$y = 3$ in (3)

$$x + 3 = 15$$

$$x = 15 - 3$$

$$\boxed{x = 12}$$

Hence, $BD = 12$, $DC = 3$.

Example 4.34

In a garden containing several trees, three particular trees P, Q, R are located in the following way. $BP = 2cm$, $CQ = 3m$, $RA = 10m$, $PC = 6m$, $QA = 5m$. $RB = 2m$, where A, B, C are points such that P lies on BC, Q lies on AC and R lies on AB. Check whether the trees P, Q, R lies on a same straight line.

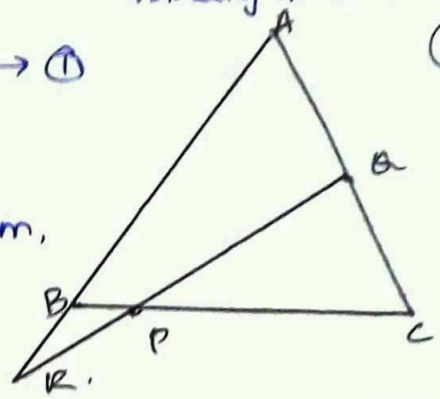
Sol

By Meanlaus's theorem,

The trees P, Q, R will be collinear (lies on same straight line).

(133)

If $\frac{BP}{PC} \times \frac{CQ}{QA} \times \frac{RA}{RB} = 1 \longrightarrow (1)$



Given:

BP = 2m, CQ = 3m, RA = 10m,

PC = 6m, QA = 5m, RB = 2m

Sub the values in (1)

$$\frac{2}{6} \times \frac{3}{5} \times \frac{10}{2} = \frac{2}{2} = 1$$

Hence, the trees P, Q, R lies on the same straight line.

Xth HAND WRITTEN MATERIALS

CHAPTER – 5 (Examples)

COORDINATE GEOMETRY

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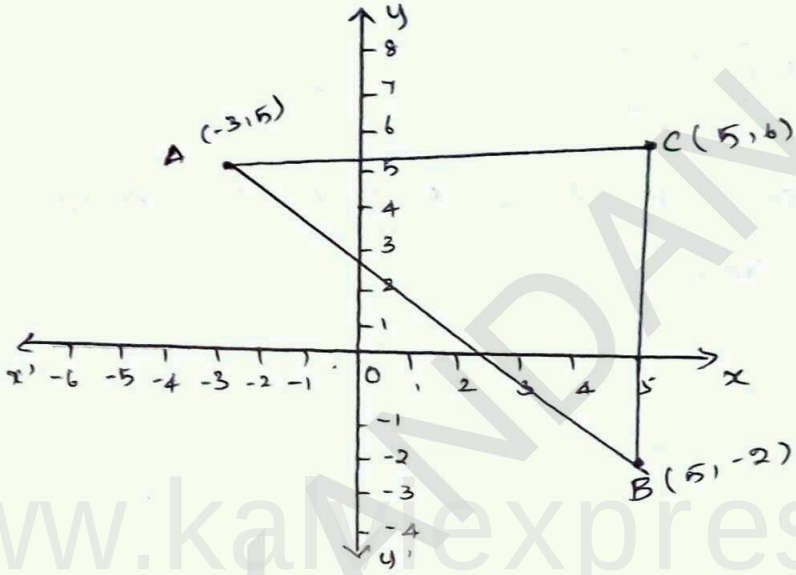
CHAPTER - 5

COORDINATE GEOMETRY

Example 5.1

Find the area of the triangle whose vertices are (-3,5), (5,6) and (+5,-2).



Sol

Plot point in the rough diagram and take them in counter-wise clock order.

Let the vertices $A(-3,5)$, $B(5,-2)$, $C(5,6)$

$\downarrow$ $\downarrow$ $\downarrow$

(x_1, y_1) (x_2, y_2) (x_3, y_3)

The area of ΔABC is

$$= \frac{1}{2}\{(x_1y_2 + y_3x_2 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\}.$$

$$= \frac{1}{2}\{(6+30+25) - (25-10-18)\}$$

$$= \frac{1}{2}\{61+3\}$$

$$= \frac{1}{2}\{64\}$$

$$= 32 \text{ sq. units.}$$

Example 5.2.

Show that the points $P(-1.5, 3)$, $Q(6, -2)$, $R(-3, 4)$ are collinear.

Sol

The points are $P(\overset{x_1}{-1.5}, \overset{y_1}{3})$, $Q(\overset{x_2}{6}, \overset{y_2}{-2})$, $R(\overset{x_3}{-3}, \overset{y_3}{4})$

Area of $\Delta PQR = \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\}$

$= \frac{1}{2}\{((-1.5 \times -2) + (6 \times 4) + (-3)(3)) - ((6)(3) + (-3)(-2) + (-1.5)(4))\}$

$= \frac{1}{2}\{(3 + 24 - 9) - (18 + 6 - 6)\}$

$= \frac{1}{2}\{18 - 18\} = 0$

$\therefore$ The given points are collinear.

Example 5.3

If the area of the triangle formed by the vertices $A(-1, 2)$, $B(k, -2)$ and $C(7, 4)$ (taken in order) is 22 sq. units, find the value of k.

Sol

The vertices are $A(\overset{x_1}{-1}, \overset{y_1}{2})$, $B(\overset{x_2}{k}, \overset{y_2}{-2})$, $C(\overset{x_3}{7}, \overset{y_3}{4})$

Area of triangle ABC is 22 sq. units.

$\frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\} = 22.$

$\frac{1}{2}\{((-1)(-2) + (k)(4) + (7)(2)) - ((k)(2) + (7)(-2) + (-1)(4))\} = 22.$

$\frac{1}{2}\{(2 + 4k + 14) - (2k - 14 - 4)\} = 22$

$\frac{1}{2}\{(16 + 4k) - (2k - 18)\} = 22$

$\frac{1}{2}\{16 + 4k - 2k + 18\} = 22$

$\frac{1}{2}\{2k + 34\} = 22$

$2k + 34 = 44$

$$2k = 44 - 34$$

$$2k = 10$$

$$k = 10/2$$

$$\boxed{k = 5}$$

Example 5.4

If the points $P(-1,-4)$, $Q(b,c)$ and $R(5,-1)$ are collinear and if $2b+c=4$, then find the values of b and c.

Sol Since the three points $P(\overset{x_1}{-1},\overset{y_1}{-4})$, $Q(\overset{x_2}{b},\overset{y_2}{c})$ and $R(\overset{x_3}{5},\overset{y_3}{-1})$ are collinear

Area of triangle PQR = 0

$$\frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2) + (x_1y_3)\} = 0$$

$$\frac{1}{2}\{((-1)(c) + (b)(-1) + (5)(-4)) - ((b)(-4) + (5)(c) + (-1)(-1))\} = 0$$

$$\frac{1}{2}\{(-c-b-20) - (-4b+5c+1)\} = 0$$

$$\frac{1}{2}\{(-c-b-20) + 4b - 5c - 1 = 0$$

$$-c-b-20+4b-5c-1 = 0$$

$$+3b-6c-21 = 0$$

$$b-2c = 7 \rightarrow (1)$$

$$2b+c = 4 \rightarrow (2)$$

$(1) \times (2) \Rightarrow$ $2b - 4c = 14$

$(2) \Rightarrow$ $2b + c = 4$

$(-) \quad (-) \quad (-)$

$$-5c = 10$$

$$c = 10/-5$$

$$\boxed{c = -2}$$

$c = -2$ in ②

$$2b - 2 = 4$$

$$2b = 4 + 2$$

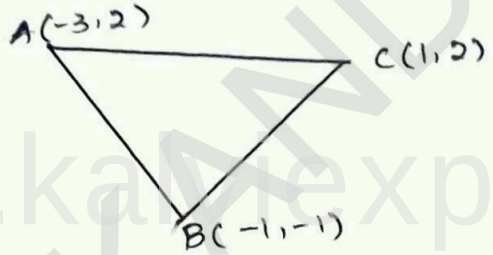
$$2b = 6$$

$$b = 6/2$$

$$\boxed{b = 3}$$

Example 5.5

The floor of a hall is covered with identical tiles which are in the shapes of triangle. One such triangle has the vertices at $(-3, 2)$, $(-1, -1)$ and $(1, 2)$. The floor of the hall is completely covered by 110 tiles, find the area of the floor.



Sol

vertices of one triangular tiles are at

$(-3, 2)$, $(-1, -1)$, $(1, 2)$

Area of this tiles $= \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\}$

$$= \frac{1}{2}\{((-3)(-1) + (-1)(2) + (1)(+2)) - ((-1)(2) + (1)(-1) + (-3)(2))\}$$

$$= \frac{1}{2}\{(3 - 2 + 2) - (-2 - 1 - 6)\}$$

$$= \frac{1}{2}\{3 + 9\} = \frac{1}{2} \times 12 = 6 \text{ sq. units.}$$

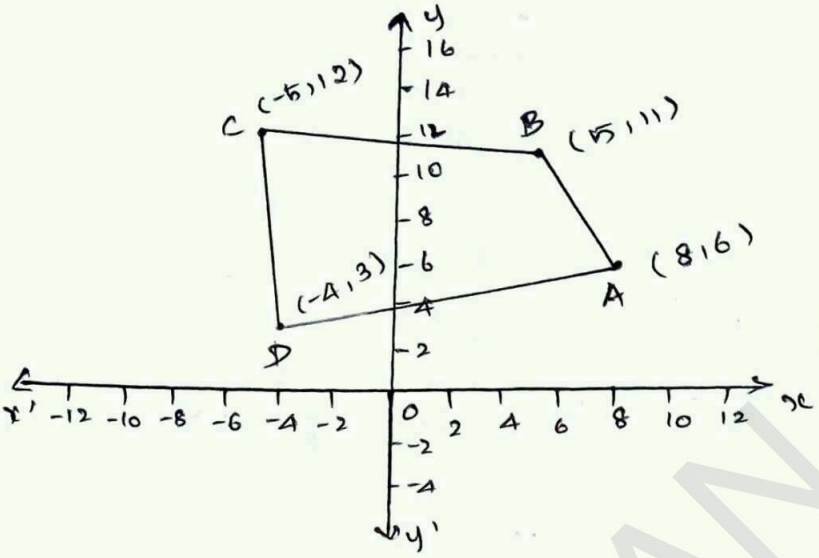
Since, the floor is covered by 110 triangle shaped identical tiles,

Area of floor $= 110 \times 6 = 660$ sq. units.

Example 5.6

Find the area of quadrilateral formed by the points (8, 6), (5, 11), (−5, 12) and (−4, 3).

Sol



Before determining the area of quadrilateral, plot the vertices of a graph.

Let the vertices be $A(\overset{x_1\ y_1}{8, 6})$, $B(\overset{x_2\ y_2}{5, 11})$, $C(\overset{x_3\ y_3}{-5, 12})$ and $D(\overset{x_4\ y_4}{-4, 3})$.

∴ Area of quadrilateral ABCD

$$= \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (x_2y_1 + x_3y_2 + x_4y_3 + x_1y_4)\}$$

$$= \frac{1}{2}\{((8)(11) + (5)(12) + (-5)(3) + (-4)(6)) - ((5)(6) + (-5)(11) + (-4)(12) + (8)(3))\}$$

$$= \frac{1}{2}\{(88 + 60 - 15 - 24) - (30 - 55 - 48 + 24)\}$$

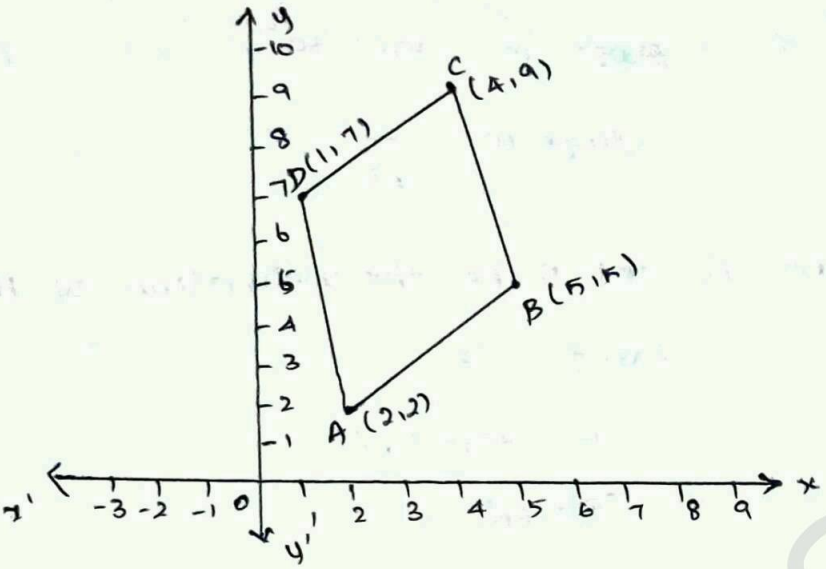
$$= \frac{1}{2}\{109 + 49\}$$

$$= \frac{1}{2}\{158\} = 79 \text{ sq. units.}$$

Example 5.7

The given diagram shows a plan for constructing a new parking lot at a campus. It is estimated that such construction would cost ₹ 1300 per square feet. What will be the total cost for making the parking lot?

Sol



Sol

The parking lot is a quadrilateral whose vertices are at A(2, 2), B(5, 5), C(4, 9), D(1, 7).

$$\text{Area of parking lot} = \frac{1}{2}\begin{vmatrix} 2 & 5 & 4 & 1 & 2 \\ 2 & 5 & 9 & 7 & 2 \end{vmatrix} \text{ sq. units.}$$

$$= \frac{1}{2}\{(10+45+28+2) - (10+20+9+14)\}$$

$$= \frac{1}{2}\{85 - 53\}$$

$$= \frac{1}{2}(32) = 16 \text{ sq. units.}$$

Area of parking lot = = 16 sq. feets.

Construction rate per square feet = ₹ 1300

Total cost for constructing the parking lot } = 16 × 1300

= ₹20800

Example 5.8

(i) What is the slope of a line whose inclination is 30°?

(ii) What is the inclination of a line whose slope is $\sqrt{3}$?

Sol (i) Here $\theta = 30°$

Slope $m = \tan\theta$

$\therefore$ slope $m = \tan 30^\circ$

slope $m = \frac{1}{\sqrt{3}}$

(ii) $m = \sqrt{3}$, let θ be the inclination of the line

$\tan\theta = \sqrt{3}$

$\theta = \tan(\sqrt{3})$

$\theta = 60^\circ$

Example 5.9

Find the slope of a line joining the given points.

(i) $(-6,1)$ and $(-3,2)$ (ii) $(-1/3, 1/2)$ and $(2/7, 3/7)$

(iii) $(14,10)$ and $(14,-6)$

Sol

(i) $(-6,1)$ and $(-3,2)$

The slope $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{2-1}{-3+6} = \frac{1}{3}$

(ii) $\left(\frac{-1}{3}, \frac{1}{2}\right)$ and $\left(\frac{2}{7}, \frac{3}{7}\right)$

Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{3}{7} - \frac{1}{2}}{\frac{2}{7} + \frac{1}{3}}$

$= \frac{\frac{6-7}{14}}{\frac{6+7}{21}} = \frac{\frac{-1}{14}}{\frac{13}{21}}$

$= \frac{-1}{14} \times \frac{21}{13} = \frac{-3}{26}$.

(iii) (14, 10) and (14, -6)

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-6-10}{14-14} = \frac{-16}{0}$$

∴ The slope is undefined.

Example 5.10

The line r passes through the points (-2, 2) and (5, 8) and the line s passes through the points (-8, 7) and (-2, 0). Is the line r perpendicular to s?

Sol

$$\text{The slope } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{The slope line r is } m_1 = \frac{8-2}{5+2} = \frac{6}{7}$$

$$\text{The slope line s is } m_2 = \frac{0-7}{-2+8} = \frac{-7}{6}$$

$$\text{The product of slope} = m_1 \times m_2 = \frac{6}{7} \times \frac{-7}{6} = -1$$

$$m_1 m_2 = -1$$

∴ the line r is perpendicular to s.

Example 5.11

The line p passes through the points (3, -2), (12, 4) and the line q passes through the points (6, -2) and (12, 2). is p parallel to q?

Sol

$$\text{The slope } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{The slope of line p is } m_1 = \frac{4+2}{12-3} = \frac{6}{9} = \frac{2}{3}$$

The slope line q is $m_2 = \frac{2+2}{12-6} = \frac{4}{6} = \frac{2}{3}$

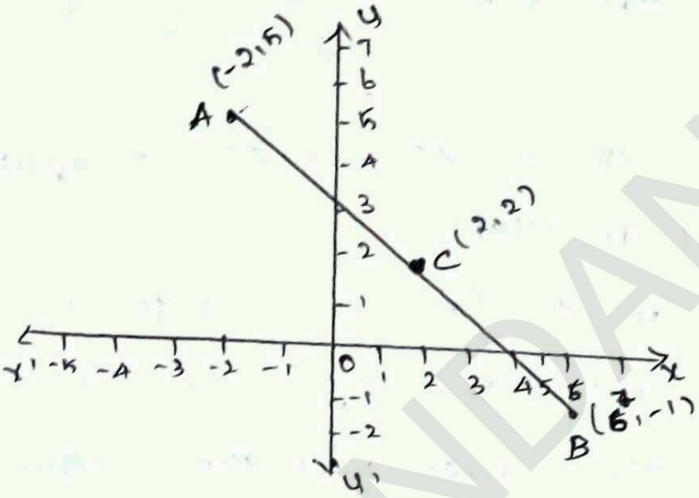
Thus, slope of line p = slope of line q

$\therefore$ Line p is parallel to the line q.

Example 5.12.

Show that the points (-2, 5), (6, -1) and (2, 2) are collinear.

Sol



The vertices A (-2, 5), B(6, -1) and C (2, 2)

Slope of AB $= \frac{-1-5}{6+2} = \frac{-6}{8} = \frac{-3}{4}$

Slope of BC $= \frac{2+1}{2-6} = \frac{3}{-4} = \frac{-3}{4}$

Slope of AB = Slope of BC

$\therefore$ The points A, B, C all lies in a same straight line.

Hence the points are A, B, C are collinear.

Example 5.13

Let A(1, -2), B(6, -2), C(5, 1) and D(2, 1) be four points.

(i) Find the slope of the line segment (a) AB (b) CD

(ii) Find the slope of the line segment (a) BC (b) AD

(iii) what can you deduce from your answer.

Sol (i) (a) Slope of AB $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2+2}{6-1} = 0$

(b) Slope of CD $= \frac{1-1}{2-5} = \frac{0}{-3} = 0$

(ii) (a) Slope of BC $= \frac{1+2}{5-6} = \frac{3}{-1} = -3$

(b) Slope of AD $= \frac{1+2}{2-1} = \frac{3}{1} = 3$

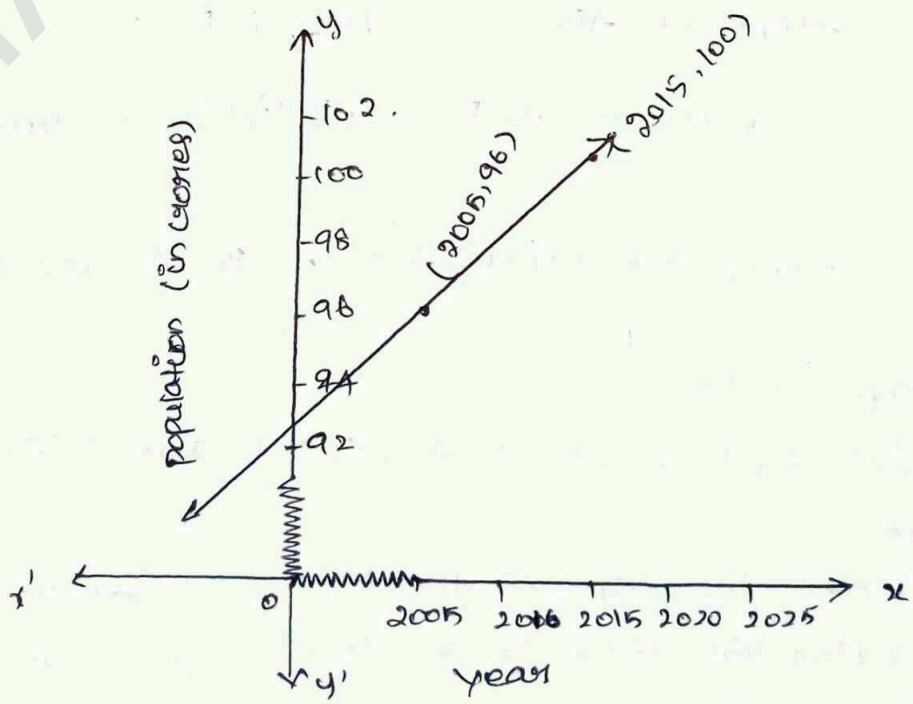
(iii) The slope of AB and CD are equal so, AB, CD are parallel.

Similarly, the lines AD and BC are not parallel, since their slopes are not equal.

So, we can deduce that the quadrilateral ABCD is a trapezium.

Example 5.14

Consider the graph representing growth of population (in crores). Find the slope of the line AB and hence estimate the population in the year 2030?



Sol The points A(2005, 96), B(2015, 100) are on the line AB.

Slope of AB $= \frac{100-96}{2015-2005} = \frac{4}{10} = \frac{2}{5}$

Let the growth of population in 2030 be K crores.

Assuming that the point C(2030, K) is on AB.

we have, slope of AC = slope of AB

$$\frac{K-96}{2030-2005} = \frac{2}{5}$$

$$\frac{K-96}{25} = \frac{2}{5}$$

$$\frac{5(K-96)}{25} = 2$$

$$K-96 = 10$$

$$K = 10+96$$

$$\boxed{K = 106}$$

Hence, the estimated population in 2030 = 106 crores.

Example 5.15

without using pythagoras theorem, show that the points (1, -4), (2, -3) and (4, -7) form a right angle triangle.

Sol Let the given points be A(1, -4), B(2, -3) and C(4, -7)

The slope of AB $= \frac{-3+4}{2-1} = \frac{1}{1} = 1$

The slope of BC $= \frac{-7+3}{4-2} = \frac{-4}{2} = -2$

The slope of AC $= \frac{-7+4}{4-1} = \frac{-3}{3} = -1$

Slope of AB × Slope of AC = (1)(−1) = −1

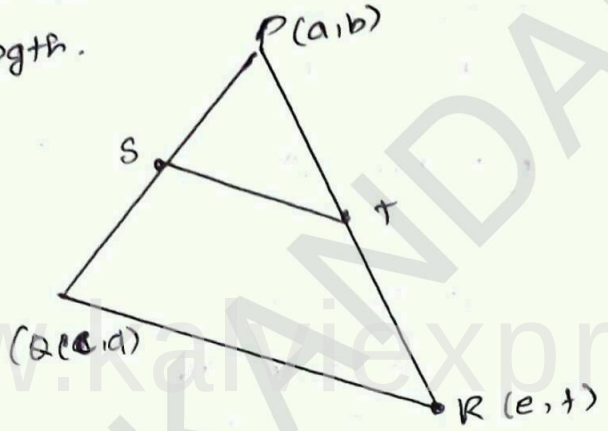
AB is perpendicular to AC

$\angle A = 90^\circ$

$\therefore$ ΔABC is a right angled triangle.

Example 5.16

Prove analytically that the line segment joining the mid points of two sides of a triangle is parallel to the third sides and is equal to half of its length.



Sol

Let P(a,b), Q(c,d) and R(e,f) be the vertices of a triangle.

Let S be the mid point of PQ and T be the midpoint of PR.

$$S = \left(\frac{a+c}{2}, \frac{b+d}{2}\right) \text{ and } T = \left(\frac{a+e}{2}, \frac{b+f}{2}\right)$$

$$\text{Slope of } ST = \frac{\frac{b+f}{2} - \frac{b+d}{2}}{\frac{a+e}{2} - \frac{a+c}{2}} = \frac{\frac{b+f-b-d}{2}}{\frac{a+e-a-c}{2}}$$

$$\text{Slope of } ST = \frac{f-d}{e-c}$$

$$\text{Slope of } QR = \frac{f-d}{e-c}$$

$\therefore$ ST is parallel to QR. (Since, their slopes are equal)

$$ST = \sqrt{\left(\frac{a+e}{2} - \frac{a+c}{2}\right)^2 + \left(\frac{b+f}{2} - \frac{b+d}{2}\right)^2}$$

$$ST = \sqrt{\left(\frac{a+e-a-c}{2}\right)^2 + \left(\frac{b+f-b-d}{2}\right)^2}$$

$$= \frac{1}{2}\sqrt{(e-c)^2 + (f-d)^2}$$

$$ST = \frac{1}{2} QR$$

Thus ST is parallel to QR and half of it.

Example 5.17

Find the equation of a straight line passing through (5, 7) and is (i) parallel to X axis (ii) parallel to Y axis.

Sol

(i) The equation of any straight line parallel to X axis is $y = b$

Since it passes through (5, 7), $b = 7$

(ii) The equation of any straight line parallel to Y-axis is $x = c$

Since it passes through (5, 7), $c = 5$

$\therefore$ The required equation of the line is $x = 5$

Example 5.18

Find the equation of a straight line whose

(i) slope is 5 and y intercept is -9

(ii) Inclination is 45° and y intercept is 11

Sol (i) Slope = 5, y intercept, c = -9

∴ Equation of a straight line is

$$y = mx + c$$

$$y = 5x - 9$$

$$5x - y - 9 = 0$$

(ii) Given, $\theta = 45°$, y intercept, $c = 11$

Slope $m = \tan\theta$

$$m = \tan 45°$$

$$m = 1$$

∴ Equation of the straight line is of the form

$$y = mx + c$$

$$y = x + 11$$

$$x - y + 11 = 0$$

Example 5.19.

Calculate the slope and y intercept of the straight line $8x - 7y + 6 = 0$

Sol Equation of the given straight line is

$$8x - 7y + 6 = 0$$

$7y = 8x + 6$ (Bringing it to the form $y = mx + c$)

$$y = \frac{8}{7}x + \frac{6}{7} \longrightarrow (1)$$

comparing (1) with $y = mx + c$

Slope $m = \frac{8}{7}$ and y intercept $c = \frac{6}{7}$

Example 5.20.

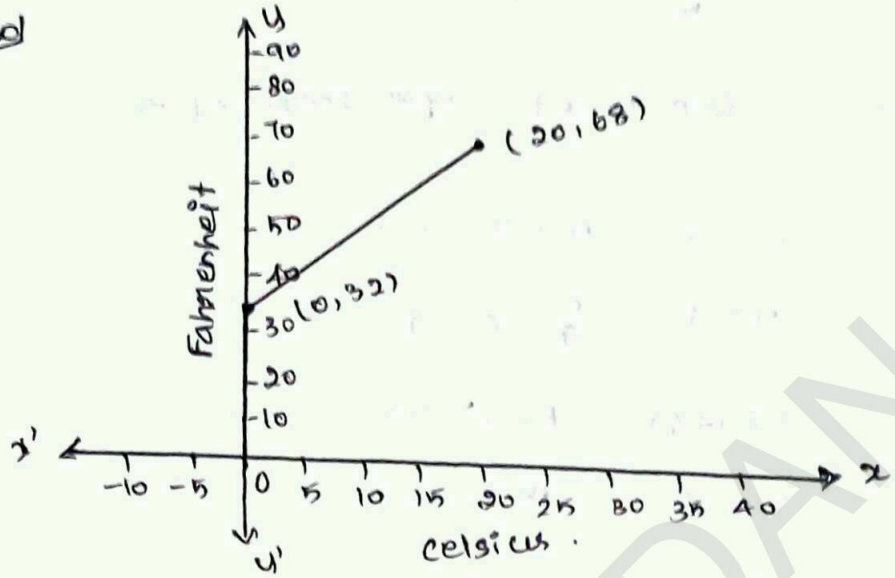
The graph relates temperatures y (in Fahrenheit degree) to temperature x (in celsius degree).

a) Find the slope and y intercept

b) Write an equation of the line

c) What is the mean temperature of the earth in Fahrenheit degree if its mean temperature is 25° celsius ?

Sol



a) From the figure, slope $= \dfrac{\text{change in y coordinate}}{\text{change in x coordinate}}$

(0, 32), (20, 68)

$$\text{slope} = \frac{68-32}{20-0} = \frac{36}{20} = \frac{9}{5} = 1.8.$$

The line cross the y axis at (0, 32)

So the slope is $\frac{9}{5}$ and y intercept is 32.

(b) use the slope and y-intercept to write an equation

The equation is $y = \frac{9}{5}x + 32$.

$\left[F = \frac{9}{5}C + 32\right]$

(c) In celsius, the mean temperature of the earth is 25°. to find the mean temperature in Fahrenheit, we find the value of y when $x = 25$

$$y = \frac{9}{5}x + 32$$

$$y = \frac{9}{5}(25) + 32$$

$$y = 45 + 32$$

$$y = 77$$

∴ The mean temperature of the earth is 77°F

Example 5.21

Find the equation of a line passing through the point (3, -4) and having slope $\frac{-5}{7}$

Sol Given: $(x_1, y_1) = (3, -4)$

$$m = \frac{-5}{7}$$

The equation of the point slope form of the straight line is

$$y - y_1 = m(x - x_1)$$

$$y + 4 = \frac{-5}{7}(x - 3)$$

$$7(y+4) = -5(x-3)$$

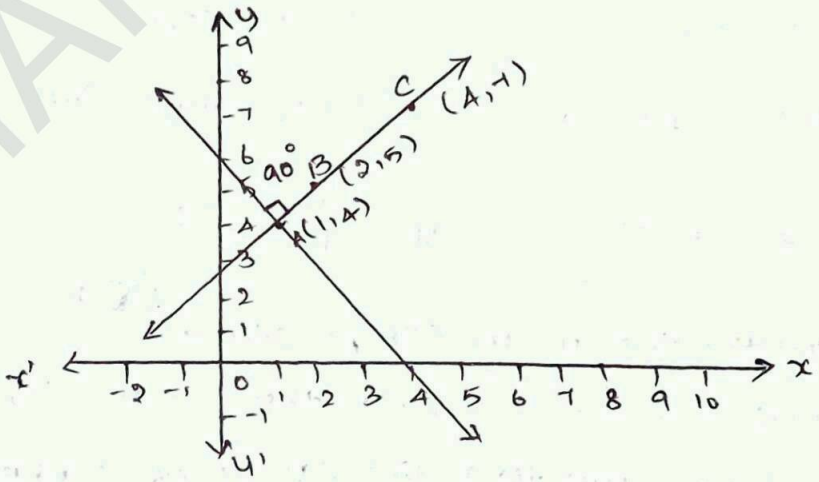
$$7y + 28 = -15x + 15$$

$$5x + 7y + 28 - 15 = 0$$

$$5x + 7y + 13 = 0$$

Example 5.22.

Find the equation of a line passing through the point A(1, 4) and perpendicular to the joining points (2, 5) and (4, 7).



Sol Let the given points A(1, 4), B(2, 5) and C(4, 7)

Slope of the line BC $= \frac{7-5}{4-2} = \frac{2}{2} = 1$

Let m be the slope of the required line.

Since, the required line is perpendicular to BC

$$m \times 1 = -1$$

(150)

$$m = -1$$

The required line also pass through the point A(1,4)

The equation of the required straight line is

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -1(x - 1)$$

$$y - 4 = -x + 1$$

$$x + y - 4 - 1 = 0$$

$$x + y - 5 = 0$$

Example 5.23:

Find the equation of a straight line passing through (5,-3) and (7,-4).

Sol The equation of a straight line passing through the two points (x_1, y_1) and (x_2, y_2) is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Substitute the points,

$$\frac{y+3}{-4+3} = \frac{x-5}{7-5}$$

$$\frac{y+3}{-1} = \frac{x-5}{2}$$

$$2(y+3) = -1(x-5)$$

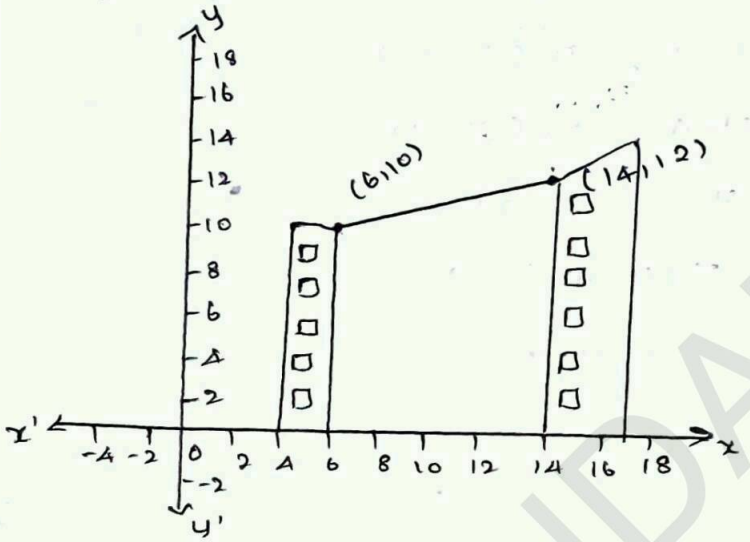
$$2y + 6 = -x + 5$$

$$x + 2y + 6 - 5 = 0$$

$$x + 2y + 1 = 0$$

Example 5.24

Two buildings of different height are located at opposite sides of each other. If a heavy rod is attached joining the terrace of the buildings from (6,10) to (14,12), find the equation of the rod joining the buildings?



Sol

Let A (6,10), B (14,12) be the point denoting the terrace of the buildings.

The equation of the rod is the equation of the straight line passing through A (6,10) and B (14,12).

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-10}{12-10} = \frac{x-6}{14-6}$$

$$\frac{y-10}{2} = \frac{x-6}{8}$$

$$8(y-10) = 2(x-6)$$

$$8y-80 = 2x-12$$

$$2x-8y-12+80=0$$

$$2x-8y+68=0$$

$$x-4y+34=0$$

$\therefore$ Equation of the rod is $x - 4y + 34 = 0$

Example 5.25

Find the equation of a line passing through (5, 7) and makes intercepts on the axes equal in magnitudes but opposite in sign.

Sol) Let the x intercept be 'a' and y intercept be '-a'

The equation of the line in intercept form

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{a} + \frac{y}{-a} = 1 \quad (\text{Here } b = -a)$$

$$\frac{x - y}{a} = 1$$

$$x - y = a \rightarrow (1)$$

Since (1) pass through (5, 7)

$$5 - 7 = a$$

$$\boxed{a = -2}$$

The required equation of the straight line

$$x - y = -2$$

$$x - y + 2 = 0$$

Example 5.26

Find the intercepts made by the line $4x - 9y + 36 = 0$ on the coordinate axes.

Sol) Equation of the given line is $4x - 9y + 36 = 0$

$$4x - 9y = -36$$

$\div$ by -36

$$\frac{4x}{-36} - \frac{9y}{-36} = \frac{-36}{-36}$$

$$\frac{x}{-9} + \frac{y}{4} = 1 \rightarrow (1)$$

Comparing ① with intercept form.

x intercept $a = -9$

y intercept $b = 4$

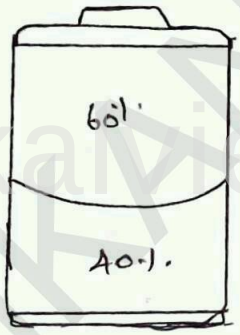
Example 5.27

A mobile phone is put to use when the battery power is 100%. the percent of battery power 'y' (in decimal) remaining after using the mobile phone for x hours is assumed as $y = -0.25x + 1$

(i) Find the number of hours elapsed if the battery power is 40%.

(ii) How much times does it take so that the battery has no power ?

Sol



Sol (i) To find the time when the battery power is 40% we have to take $y = 0.40$

$$0.40 = -0.25x + 1$$

$$0.25x = 1 - 0.40$$

$$0.25x = 0.60$$

$$x = \frac{0.60}{0.25} = 2.4 \text{ hours.}$$

(ii) If the battery power is 0 then $y = 0$

$$0 = -0.25x + 1$$

$$0.25x = 1$$

$$x = \frac{1}{0.25} = \frac{100}{25} = 4 \text{ hours.}$$

$\therefore$ After 4 hours, the battery of the mobile phone will have no power.

Example 5.28

A line makes positive intercepts on coordinate axes whose sum is 7 and it passes through (-3, 8). Find its equation.

Sol

If a and b are the intercepts then

$$a+b = 7$$

$$b = 7-a$$

By Intercept form

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{a} + \frac{y}{7-a} = 1$$

As this line passes through the point (-3, 8), we have

$$\frac{-3}{a} + \frac{8}{7-a} = 1$$

$$\frac{-3(7-a)+8a}{(7-a)a} = 1$$

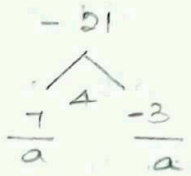
$$-21+3a+8a = 7a-a^2$$

$$a^2-7a+3a+8a-21=0$$

$$a^2+4a-21=0$$

$$(a+7)(a-3)=0$$

$$a=3, \quad a=-7$$



Since a is positive, $a=3$ and $b=7-a=7-3=4$

Hence $\frac{x}{3} + \frac{y}{4} = 1$

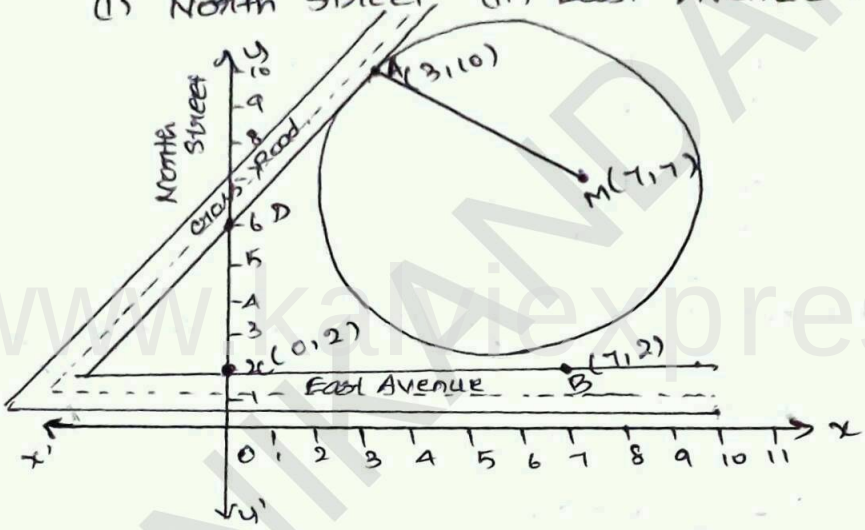
$\therefore$ $4x+3y=12$ is the required equation

Example 5.29

A circular garden is bounded by East Avenue and cross road. Cross Road intersects North street at D and East Avenue at E. AD is a tangential to the circular garden at A (3,10). using the figure.

a) Find the equation of
   (i) East Avenue
   (ii) North street
   (iii) Cross Road.

b) Where does the Cross Road intersect?
   (i) North street (ii) East Avenue.



Sol (a)

(i) East Avenue is the straight line joining C(0,2) and B(7,2). thus the equation of East Avenue is obtained by using two-point form which is

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-2}{2-2} = \frac{x-0}{7-0}$$

$$\frac{y-2}{0} = \frac{x}{7}$$

$$7y-14=0$$

$$y = 14/7 = 2$$

(ii) Since the point D lie vertically above C(0,2). the x coordinate of D is 0. Since any point on North street has x coordinate value 0. (1158)

The equation of North street is $x=0$

(iii) To find equation of Cross Road.

Centre of circular garden M is at (7,7), A is (3,10)

Find slope of MA, which we call m_1

$$m_1 = \frac{10-7}{3-7} = \frac{-3}{4}$$

Since the cross road is perpendicular to MA. If m_2 is the slope of the cross road.

$$m_1 m_2 = -1$$

$$-\frac{3}{4} \times m_2 = -1$$

$$-3m_2 = -4$$

$$m_2 = \frac{-4}{-3} = \frac{4}{3}$$

Now, the cross road has slope $\frac{4}{3}$ and it passes through the point A(3,10)

the equation of the cross road is $y-10 = \frac{4}{3}(x-3)$

$$3(y-10) = 4x-12$$

$$3y-30 = 4x-12$$

$$4x-3y-12+30=0$$

$$4x-3y+18=0$$

(b)

(i) If D is (0,K) then D is a point on the cross road.

substitute, $x=0$, $y=K$ in the equation of cross road.

$$0-3K+18=0$$

$$3K=18$$

$$\boxed{K=6}$$

$\therefore$ D is (0,6)

(ii) to find, E let E be $(a, 2)$

Put $y = 2$ in the equation of cross road.

$$4a - 6 + 18 = 0$$

$$4a = -12$$

$$\boxed{a = -3}$$

$\therefore$ The point E is $(-3, 2)$.

Thus, the cross road meets the North street at D$(0, 6)$ and East Avenue at E$(-3, 2)$

Example 5.30

Find the slope of the straight line $6x + 8y + 7 = 0$

Sol Given: $6x + 8y + 7 = 0$

$$\text{slope } m = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-6}{8} = -\frac{3}{4}$$

$\therefore$ The slope of the straight line is $-\frac{3}{4}$.

Example 5.31

Find the slope of the line which is

(i) parallel to $3x - 7y = 11$ (ii) perpendicular to $2x - 3y + 8 = 0$

Sol (i) Given straight line is $3x - 7y = 11$

$$3x - 7y - 11 = 0$$

$$\text{Slope } m = -\frac{\text{coefficient of } x}{\text{coefficient of } y} = \cancel{\frac{7}{3}}$$

$$= \frac{-3}{-7} = \frac{3}{7}$$

Since parallel line have same slopes, slopes of any line parallel to

$3x - 7y = 11$ is $\frac{3}{7}$

(ii) Given straight line is $2x - 3y + 8 = 0$

$$\text{slope } m = \frac{-2}{-3} = \frac{2}{3}$$

Since product of slope is -1 for perpendicular lines, slope of any line perpendicular to $2x - 3y + 8 = 0$ is -1

$$\frac{2}{3} = -1 \Rightarrow \frac{-3}{2}$$

Example 5.32.

Show that the straight lines $2x + 3y - 8 = 0$ and $4x + 6y + 18 = 0$ are parallel.

Sol

Slope of the straight line $2x + 3y - 8 = 0$

$$m_1 = \frac{-\text{coefficient of } x}{\text{coefficient of } y}$$

$$m_1 = \frac{-2}{3}$$

Slope of the straight line $4x + 6y + 18 = 0$

$$m_2 = \frac{-4}{+6} = \frac{-2}{3}$$

$$m_1 = m_2$$

Slope are equal. Hence, the two straight lines are parallel.

Example 5.33

Show that the straight lines $x - 2y + 3 = 0$ and $6x + 3y + 8 = 0$ are perpendicular.

Sol

Slope of the straight line $x - 2y + 3 = 0$

$$m = \frac{-1}{-2} = \frac{1}{2}$$

Slope of the straight line $6x + 3y + 8 = 0$

$$m_2 = -\frac{6}{3} = -2$$

$$m_1 \times m_2 = \frac{1}{2} \times -2 = -1$$

$\therefore$ Two straight lines are perpendicular.

Example 5.34

Find the equation of the straight line which is parallel to the line $3x - 7y = 12$ and passing through the point $(6, 4)$.

Sol

Equation of the straight line, parallel to $3x - 7y - 12 = 0$ is $3x - 7y + K = 0$

since it passes through the point $(6, 4)$

$$3(6) - 7(4) + K = 0$$

$$18 - 28 + K = 0$$

$$K = 10$$

$\therefore$ Equation of the required straight line is $3x - 7y + 10 = 0$.

Example 5.35

Find the equation of the straight line perpendicular to the line $y = \frac{4}{3}x - 7$ and passing through the point $(7, -1)$.

Sol

The equation $y = \frac{4x}{3} - 7$ can be written as

$4x - 3y - 21 = 0$

Equation of the straight line perpendicular to $4x - 3y - 21 = 0$ is $3x + 4y + K = 0$

since it passes through the point $(7, -1)$

$$3(7) + 4(-1) + K = 0$$

$$21 - 4 + K = 0$$

$$\boxed{K = -17}$$

$\therefore$ Equation of the required straight line is

$$3x + 4y - 17 = 0$$

Example 5.36

Find the equation of a straight line parallel to Y axis and passing through the point of intersection of the lines $4x+5y=13$ and $x-8y+9=0$

Sol Given lines

$$4x+5y=13 \Rightarrow 4x+5y-13=0 \rightarrow (1)$$
$$x-8y=-9 \Rightarrow 4x-8y+9=0 \rightarrow (2)$$

to find point of intersection, solve (1) & (2)

$$\begin{array}{ccccccc} x & & y & & 1 & & \\ 5 & & -13 & & 4 & & 5 \\ -8 & & 9 & & 1 & & -8 \end{array}$$

$$\frac{x}{45-104} = \frac{y}{-13-36} = \frac{1}{-32-5}$$

$$\frac{x}{-59} = \frac{y}{-49} = \frac{1}{-37}$$

$$x = \frac{59}{37}, \quad y = \frac{49}{37}$$

$\therefore$ The point of intersection $(x,y) = \left(\frac{59}{37}, \frac{49}{37}\right)$

The equation of lines parallel to Y axis is $x=c$

It passes through $(x,y) = \left(\frac{59}{37}, \frac{49}{37}\right)$

$$\therefore c = \frac{59}{37}$$

The equation of the line is $x = \frac{59}{37}$

$$37x - 59 = 0$$

Example 5.37

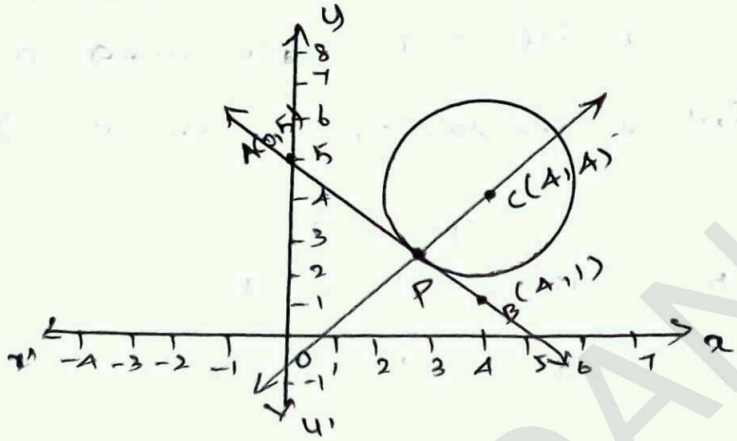
the line joining the points $A(0,5)$ and $B(4,1)$ is a tangent to a circle whose centre c is at the

point (4, 4) find

(i) the equation of the line AB

(ii) The equation of the line through C which is perpendicular to the line AB

(iii) The coordinates of the point of contact of tangent line AB with the circle.



Sol (i) Equation of line AB, A(0,5) and (B (4,1)

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-5}{1-5} = \frac{x-0}{4-0}$$

$$4(y-5) = -4x$$

$$4y-20 = -4x$$

$$4x+4y-20=0$$

$$x+y-5=0$$

(ii) The equation of a line which is perpendicular to the line AB: $x+y-5=0$ is $x-y+k=0$

Since it passes through the point (4, 4),

$$4-4+k=0$$

$$\boxed{k=0}$$

The equation of a line which is perpendicular to AB and through C is $x-y=0$

(iii) The coordinate of the point of contact P of the tangent line AB with the circle is point of intersection of lines. (163)

$$x+y-5=0 \quad \text{and} \quad x-y=0$$

Solving the equation

$$\begin{aligned} x+y-5 &= 0 \\ x-y &= 0 \\ \hline 2x-5 &= 0 \\ 2x &= 5 \\ x &= 5/2 \end{aligned}$$

$x = 5/2$ in $x-y=0$

$$\frac{5}{2} - y = 0$$

$$\frac{5}{2} = y$$

$$\therefore \quad x = \frac{5}{2}, \quad y = \frac{5}{2}$$

The coordinate of the point of contact is

$$P\left(\frac{5}{2}, \frac{5}{2}\right).$$

Xth HAND WRITTEN MATERIALS

CHAPTER – 6 (Examples)

TRIGONOMETRY

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P.G Teacher Mathematics

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CHAPTER - 6

TRIGONOMETRY

Example 6.1

Prove that $\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$

Sol

$$\tan^2\theta - \sin^2\theta = \tan^2\theta - \frac{\sin^2\theta}{\cos^2\theta} \times \cos^2\theta$$

$$= \tan^2\theta - \tan^2\theta \cos^2\theta$$

$$= \tan^2\theta (1 - \cos^2\theta)$$

$$\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$$

$[\because \sin^2\theta = 1 - \cos^2\theta]$

Example 6.2

Prove that $\frac{\sin A}{1+\cos A} = \frac{1-\cos A}{\sin A}$

Sol

$$\frac{\sin A}{1+\cos A} = \frac{\sin A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}$$

[multiplying numerator and denominator by the conjugate of $1+\cos A$]

$$= \frac{\sin A (1-\cos A)}{(1+\cos A)(1-\cos A)}$$

$$= \frac{\sin A (1-\cos A)}{1-\cos^2 A}$$

$$= \frac{\sin A (1-\cos A)}{\sin^2 A}$$

$$\therefore \frac{\sin A}{1+\cos A} = \frac{1-\cos A}{\sin A}$$

Example 6.3

prove that $1 + \frac{\cot^2\theta}{1+\operatorname{cosec}\theta} = \operatorname{cosec}\theta$

Sol

$$1 + \frac{\cot^2\theta}{1+\operatorname{cosec}\theta} = 1 + \frac{\operatorname{cosec}^2\theta - 1}{1+\operatorname{cosec}\theta}$$

$[\because \operatorname{cosec}^2\theta - 1 = \cot^2\theta]$

$$= 1 + \frac{(\operatorname{cosec}\theta+1)(\operatorname{cosec}\theta-1)}{(\operatorname{cosec}\theta+1)}$$

$$= 1 + (\operatorname{cosec}\theta - 1)$$

$$= 1 + \operatorname{cosec}\theta - 1$$

$$1 + \frac{\cot^2\theta}{1+\operatorname{cosec}\theta} = \operatorname{cosec}\theta$$

Example 6.4

prove that $\sec\theta - \cos\theta = \tan\theta\sin\theta$

Sol

$$\sec\theta - \cos\theta = \frac{1}{\cos\theta} - \cos\theta$$

$$= \frac{1-\cos^2\theta}{\cos\theta} \qquad [\because 1-\cos^2\theta = \sin^2\theta]$$

$$= \frac{\sin^2\theta}{\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} \times \sin\theta$$

$$\sec\theta - \cos\theta = \tan\theta\sin\theta$$

Example 6.5

prove that $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \operatorname{cosec}\theta + \cot\theta$

Sol

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1-\cos\theta}}$$

[multiply numerator and denomenator by the conjugate of $1-\cos\theta$]

$$= \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}}$$

$$= \frac{1+\cos\theta}{\sqrt{\sin^2\theta}}$$

[$\because \sin^2\theta + \cos^2\theta = 1$]

$$= \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \operatorname{cosec}\theta + \cot\theta$$

Example 6.6

prove that $\frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \cot\theta$

Sol

$$\frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{\cos\theta}}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta\sin\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1 - \sin\theta \times \sin\theta}{\cos\theta\sin\theta}$$

$$= \frac{1-\sin^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$\frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \cot\theta$$

Example 6.7

Prove that $\sin^2 A\cos^2 B + \cos^2 A\sin^2 B + \cos^2 A\cos^2 B + \sin^2 A\sin^2 B = 1$

Sol

$\sin^2 A\cos^2 B + \cos^2 A\sin^2 B + \cos^2 A\cos^2 B + \sin^2 A\sin^2 B$

$= \sin^2 A\cos^2 B + \sin^2 A\sin^2 B + \cos^2 A\sin^2 B + \cos^2 A\cos^2 B$

$= \sin^2 A(\cos^2 B + \sin^2 B) + \cos^2 A(\sin^2 B + \cos^2 B)$

$= \sin^2 A(1) + \cos^2 A(1)$ $\quad [\because \sin^2 B + \cos^2 B = 1]$

$= \sin^2 A + \cos^2 A$

$= 1$

$\sin^2 A\cos^2 B + \cos^2 A\sin^2 B + \cos^2 A\cos^2 B + \cancel{\cos^2}\sin^2 A\sin^2 B = 1$

Example 6.8

If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

Sol

Now, $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

squaring both sides,

$$(\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$$

$\cos^2\theta + \sin^2\theta + 2\cos^2\theta\sin^2\theta$

$$\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta = 2\cos^2\theta$$

$$2\cos^2\theta - \cos^2\theta - \sin^2\theta = 2\sin\theta\cos\theta$$

$$\cos^2\theta - \sin^2\theta = 2\sin\theta\cos\theta$$

$$(\cos\theta + \sin\theta)(\cos\theta - \sin\theta) = 2\sin\theta\cos\theta$$

$$\cos\theta - \sin\theta = \frac{2\sin\theta\cos\theta}{\cos\theta + \sin\theta}$$

$$\cos\theta - \sin\theta = \frac{2\sin\theta\cos\theta}{\sqrt{2}\cos\theta}$$

$$[\because \cos\theta + \sin\theta = \sqrt{2}\cos\theta]$$

$$\cos\theta - \sin\theta = \frac{\sqrt{2} \times \sqrt{2}\sin\theta}{\sqrt{2}}$$

$$\therefore \cos\theta - \sin\theta = \sqrt{2}\sin\theta$$

Example 6.9

prove that $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$

Sol) $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)$$

$$= \frac{(1-\sin^2\theta)(1-\cos^2\theta)(1)}{\sin^2\theta\cos^2\theta}$$

$$= \frac{(1 - \sin^2\theta - \cos^2\theta + \sin^2\theta\cos^2\theta)(1)}{\sin^2\theta\cos^2\theta}$$

$$= \frac{1-(\sin^2\theta+\cos^2\theta)+(\sin^2\theta\times\cos^2\theta)(1)}{\sin^2\theta\cos^2\theta}$$

$$= \frac{1-1+\sin^2\theta\cos^2\theta\,(1)}{\sin^2\theta\cos^2\theta}$$

$$= \frac{\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta} = 1$$

$\therefore (\operatorname{cosec}\theta-\sin\theta)(\sec\theta-\cos\theta)(\tan\theta+\cot\theta)=1$

Example 6.10

Prove that $\frac{\sin A}{1+\cos A}+\frac{\sin A}{1-\cos A}=2\operatorname{cosec}A$

Sol

$$\frac{\sin A}{1+\cos A}+\frac{\sin A}{1-\cos A}=\frac{\sin A(1-\cos A)+\sin A(1+\cos A)}{1-\cos^2 A}$$

$$=\frac{\sin A-\sin A\cos A+\sin A+\sin A\cos A}{1-\cos^2 A}$$

$$=\frac{2\sin A}{\sin^2 A} \quad [\because 1-\cos^2 A=\sin^2 A]$$

$$=\frac{2}{\sin A}=2\times\frac{1}{\sin A}$$

$$\frac{\sin A}{1+\cos A}+\frac{\sin A}{1-\cos A}=2\operatorname{cosec}A$$

Example 6.11

If $\operatorname{cosec}\theta+\cot\theta=p$, then prove that $\cos\theta=\frac{p^2-1}{p^2+1}$

Sol

$\operatorname{cosec}\theta+\cot\theta=p \longrightarrow (1)$

$\operatorname{cosec}^2\theta-\cot^2\theta=1$ (Identity)

$$(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$$

$$\operatorname{cosec}\theta - \cot\theta = \frac{1}{\operatorname{cosec}\theta + \cot\theta}$$

$$\operatorname{cosec}\theta - \cot\theta = \frac{1}{P} \rightarrow (2)$$

Adding (1) & (2)

$$\operatorname{cosec}\theta + \cot\theta + \operatorname{cosec}\theta - \cot\theta = P + \frac{1}{P}$$

$$2\operatorname{cosec}\theta = \frac{P^2+1}{P} \longrightarrow (3)$$

subtracting (2) from (1)

$$\operatorname{cosec}\theta + \cot\theta - (\operatorname{cosec}\theta - \cot\theta) = P - \frac{1}{P}$$

$$\operatorname{cosec}\theta + \cot\theta - \operatorname{cosec}\theta + \cot\theta = \frac{P^2-1}{P}$$

$$2\cot\theta = \frac{P^2-1}{P} \longrightarrow (4)$$

Dividing (4) by (3)

$$\frac{2\cot\theta}{2\operatorname{cosec}\theta} = \frac{\frac{P^2-1}{P}}{\frac{P^2+1}{P}}$$

$$\frac{2\cot\theta}{2\operatorname{cosec}\theta} = \frac{P^2-1}{P} \times \frac{P}{P^2+1}$$

$$\frac{\cot\theta}{\operatorname{cosec}\theta} = \frac{P^2-1}{P^2+1}$$

$$\frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} = \frac{P^2-1}{P^2+1}$$

$$\frac{\cos\theta}{\sin\theta} \times \frac{\sin\theta}{1} = \frac{P^2-1}{P^2+1}$$

$$\cos\theta = \frac{P^2-1}{P^2+1}$$

Example 6.12

Prove that $\tan^2 A - \tan^2 B = \dfrac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}$

Sol

$$\tan^2 A - \tan^2 B = \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}$$

$$= \frac{\sin^2 A \cos^2 B - \sin^2 B \cos^2 A}{\cos^2 A \cos^2 B}$$

$$= \frac{\sin^2 A (1 - \sin^2 B) - \sin^2 B (1 - \sin^2 A)}{\cos^2 A \cos^2 B}$$

$$= \frac{\sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B}{\cos^2 A \cos^2 B}$$

$$\tan^2 A - \tan^2 B = \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}$$

Example 6.13

Prove that $\left(\dfrac{\cos^3 A - \sin^3 A}{\cos A - \sin A}\right) - \left(\dfrac{\cos^3 A + \sin^3 A}{\cos A + \sin A}\right) = 2 \sin A \cos A$

Sol

$$\left(\frac{\cos^3 A - \sin^3 A}{\cos A - \sin A}\right) - \left(\frac{\cos^3 A + \sin^3 A}{\cos A + \sin A}\right)$$

$$= \left(\frac{(\cos A - \sin A)(\cos^2 A + \sin^2 A + \cos A \sin A)}{\cos A - \sin A}\right) -$$

$$\left(\frac{(\cos A + \sin A)(\cos^2 A + \sin^2 A - \cos A \sin A)}{\cos A + \sin A}\right)$$

$[\because \; a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$

$a^3 + b^3 = (a+b)(a^2 + b^2 - ab)]$

$$= (1 + \cos A \sin A) - (1 - \cos A \sin A)$$

$$= 1 + \cos A \sin A - 1 + \cos A \sin A$$

$$= 2 \cos A \sin A$$

$$\left(\frac{\cos^3 A - \sin^3 A}{\cos A - \sin A}\right) - \left(\frac{\cos^3 A + \sin^3 A}{\cos A + \sin A}\right) = 2 \sin A \cos A.$$

Example 6.14:

Prove that $\dfrac{\sin A}{\sec A + \tan A - 1} + \dfrac{\cos A}{\operatorname{cosec} A + \cot A - 1} = 1$

Sol

$$\frac{\sin A}{\sec A + \tan A - 1} + \frac{\cos A}{\operatorname{cosec} A + \cot A - 1}$$

$$= \frac{\sin A (\operatorname{cosec} A + \cot A - 1) + \cos A (\sec A + \tan A - 1)}{(\sec A + \tan A - 1)(\operatorname{cosec} A + \cot A - 1)}$$

$$= \frac{\sin A \operatorname{cosec} A + \sin A \cot A - \sin A + \cos A \sec A + \tan A \cos A - \cos A}{(\sec A + \tan A - 1)(\operatorname{cosec} A + \cot A - 1)}$$

$$= \frac{\sin A \times \frac{1}{\sin A} + \sin A \times \frac{\cos A}{\sin A} - \sin A + \cos A \times \frac{1}{\cos A} + \frac{\sin A}{\cos A} \times \cos A - \cos A}{\left[\frac{1}{\cos A} + \frac{\sin A}{\cos A} - 1\right]\left[\frac{1}{\sin A} + \frac{\cos A}{\sin A} - 1\right]}$$

$$= \frac{1 + \cos A - \sin A + 1 + \sin A - \cos A}{\left[\frac{1 + \sin A - \cos A}{\cos A}\right]\left[\frac{1 + \cos A - \sin A}{\sin A}\right]}$$

$$= \frac{2}{\frac{(1+\sin A-\cos A)(1+\cos A-\sin A)}{\sin A\cos A}}$$

$$= \frac{2\sin A\cos A}{[1+(\sin A-\cos A)][1-(\sin A-\cos A)]}$$

$$= \frac{2\sin A\cos A}{1-(\sin A-\cos A)^2}$$

$$= \frac{2\sin A\cos A}{1-[\sin^2 A+\cos^2 A-2\sin A\cos A]}$$

$$= \frac{2\sin A\cos A}{1-[1-2\sin A\cos A]}$$

$$= \frac{2\sin A\cos A}{1-1+2\sin A\cos A}$$

$$= \frac{2\sin A\cos A}{2\sin A\cos A}$$

$$= 1$$

$$\frac{\sin A}{\sec A+\tan A-1}+\frac{\cos A}{\operatorname{cosec} A+\cot A-1} = 1$$

Example 6.15

Show that $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2$ (1+3)

Sol

LHS:

$$\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \frac{1+\tan^2 A}{1+\frac{1}{\tan^2 A}}$$

$$= \frac{1+\tan^2 A}{\frac{\tan^2 A+1}{\tan^2 A}}$$

$$= 1+\tan^2 A \times \frac{\tan^2 A}{\tan^2 A+1}$$

$$\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \tan^2 A \longrightarrow (1)$$

RHS:

$$\left(\frac{1-\tan^2 A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$$

$$= \left(\frac{1-\tan A}{\frac{\tan A-1}{\tan A}}\right)^2$$

$$= \left(1-\tan A \times \frac{\tan A}{-(1-\tan A)}\right)^2$$

$$= (-\tan A)^2$$

$$\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A \longrightarrow (2)$$

From ① & ②,

$$\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2$$

Hence ~~pr~~ proved.

Example 6.16:

prove that $\dfrac{(1+\cot A+\tan A)(\sin A-\cos A)}{\sec^3 A-\operatorname{cosec}^3 A} = \sin^2 A\cos^2 A$

Sol

$$\frac{(1+\cot A+\tan A)(\sin A-\cos A)}{\sec^3 A-\operatorname{cosec}^3 A}$$

$$= \frac{\left(1+\frac{\cos A}{\sin A}+\frac{\sin A}{\cos A}\right)(\sin A-\cos A)}{(\sec A-\operatorname{cosec} A)(\sec^2 A+\operatorname{cosec}^2 A+\sec A\operatorname{cosec} A)}$$

$$= \frac{\left[\frac{\sin A\cos A+\cos^2 A+\sin^2 A}{\sin A\cos A}\right](\sin A-\cos A)}{(\sec A-\operatorname{cosec} A)\left[\frac{1}{\cos^2 A}+\frac{1}{\sin^2 A}+\frac{1}{\cos A}\cdot\frac{1}{\sin A}\right]}$$

$$= \frac{\frac{(\sin A\cos A+1)(\sin A-\cos A)}{\sin A\cos A}}{(\sec A-\operatorname{cosec} A)\left[\frac{\sin^2 A+\cos^2 A+\sin A\cos A}{\sin^2 A\cos^2 A}\right]}$$

$$= \frac{(\sin A\cos A+1)\left[\frac{\sin A}{\sin A\cos A}-\frac{\cos A}{\sin A\cos A}\right]}{(\sec A-\operatorname{cosec} A)\left[\frac{1+\sin A\cos A}{\sin^2 A\cos^2 A}\right]}$$

$$= \frac{(\sin A\cos A+1)\left[\frac{1}{\cos A}-\frac{1}{\sin A}\right]}{(\sec A-\operatorname{cosec} A)(1+\sin A\cos A)} \times \sin^2 A\cos^2 A$$

$$= \frac{\sin A\cos A+1\,(\sec A-\operatorname{cosec} A)}{(\sec A-\operatorname{cosec} A)(1+\sin A\cos A)} \times \sin^2 A\cos^2 A$$

$$= 1 \times \sin^2 A\cos^2 A$$

$$\frac{(1+\cot A+\tan A)(\sin A-\cos A)}{\sec^3 A-\operatorname{cosec}^3 A} = \sin^2 A\cos^2 A$$

Example 6.17

If $\frac{\cos^2\theta}{\sin\theta} = p$, $\frac{\sin^2\theta}{\cos\theta} = q$, then prove that $p^2q^2(p^2+q^2+3) = 1$

Sol.

$$\frac{\cos^2\theta}{\sin\theta} = p \longrightarrow (1)$$

$$\frac{\sin^2\theta}{\cos\theta} = q \longrightarrow (2)$$

$$p^2q^2(p^2+q^2+3) = \left(\frac{\cos^2\theta}{\sin\theta}\right)^2\left(\frac{\sin^2\theta}{\cos\theta}\right)^2\left[\left(\frac{\cos^2\theta}{\sin\theta}\right)^2+\left(\frac{\sin^2\theta}{\cos\theta}\right)^2+3\right]$$

$$= \left(\frac{\cos^4\theta}{\sin^2\theta}\right)\left(\frac{\sin^4\theta}{\cos^2\theta}\right)\left[\frac{\cos^4\theta}{\sin^2\theta}+\frac{\sin^4\theta}{\cos^2\theta}+3\right]$$

$$= (\cos^2\theta \times \sin^2\theta)\left[\frac{\cos^6\theta+\sin^6\theta+3\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}\right]$$

$$= \cos^6\theta+\sin^6\theta+3\sin^2\theta\cos^2\theta$$

$$= (\cos^2\theta)^3+(\sin^2\theta)^3+3\sin^2\theta\cos^2\theta$$

$$= [(\cos^2\theta + \sin^2\theta)^3 - 3\cos^2\theta\sin^2\theta(\cos^2\theta + \sin^2\theta)]$$

$$+ 3\sin^2\theta\cos^2\theta$$ (176)

$$= (1)^3 - 3\cos^2\theta\sin^2\theta(1) + 3\sin^2\theta\cos^2\theta$$

$$= 1 - 3\cos^2\theta\sin^2\theta + 3\cos^2\theta\sin^2\theta$$

$$= 1$$

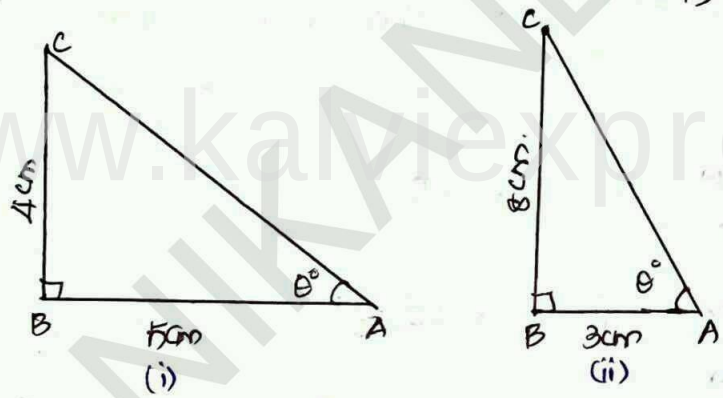
$$p^2q^2(p^2+q^2+3) = 1$$

Hence proved.

Example 6.18

Calculate the size of $\angle BAC$ in the given triangles.

($\tan 38.7^\circ = 0.8011$, $\tan 69.4^\circ = 2.6604$)



Sol

(i) In the Right angled ΔABC

$$\tan\theta = \frac{\text{opposite side}}{\text{Adjacent side}} = \frac{4}{5}$$

$$\theta = \tan^{-1}\left(\frac{4}{5}\right) = \tan^{-1}(0.8)$$

$$\theta = 38.7^\circ \quad (\because \tan 38.7^\circ = 0.8011)$$

$$\angle BAC = 38.7^\circ$$

(ii) In the Right angled ΔABC

$$\tan\theta = \frac{8}{3}$$

$$\theta = \tan^{-1}\left(\frac{8}{3}\right)$$

$= \tan^{-1}(2.66)$

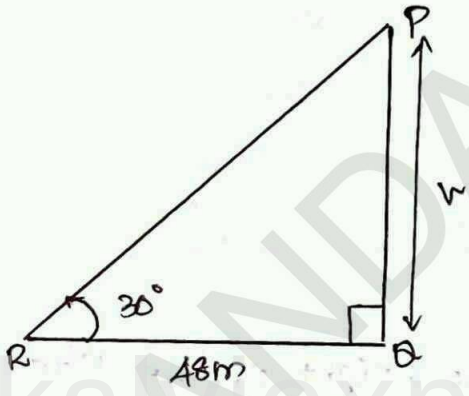
$\theta = 69.4^\circ \quad (\because \tan 69.4^\circ = 2.6604)$

$\angle BAC = 69.4^\circ$

Example 6.19

A tower stands vertically on the ground. From a point on the ground, which is 48m away from the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the power.

Sol



Let PQ be the height of the tower

Take PQ = h and QR is the distance between the tower and the point R.

In the Right angled ΔPQR, $\angle PRQ = 30^\circ$

$$\tan\theta = \frac{\text{opposite side}}{\text{Adjacent side}}$$

$$\tan\theta = \frac{PQ}{QR}$$

$$\tan 30^\circ = \frac{h}{48}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{48} \Rightarrow h = \frac{48}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

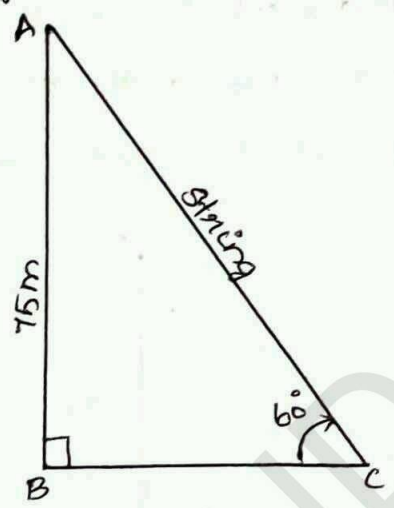
$$h = \frac{48\sqrt{3}}{3} = 16\sqrt{3}$$

$\therefore$ The height of the tower is $16\sqrt{3}$m

Example 6.20

A kite is flying at a height of 75m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string. assuming that there is no slack in the string.

Sol



Let AB be the height of the kite above the ground. Then, AB = 75.

Let AC be the length of the string

In the right angled ΔABC, $\angle ACB = 60°$

$$\sin\theta = \frac{AB}{AC}$$

$$\sin 60° = \frac{75}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{75}{AC}$$

$$AC = \frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{150\sqrt{3}}{3}$$

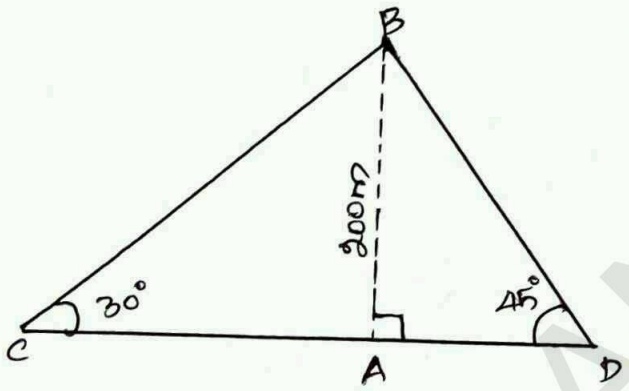
$$AC = 50\sqrt{3}$$

Hence, the length of the string is $50\sqrt{3}$ m.

Example 6.21

Two ships are sailing in the sea on either sides of a lighthouse. the angle of elevation of the top of the lighthouse as observed from the ships are 30° and 45° respectively. If the lighthouse is 200m high, find the distance between the two ships. ($\sqrt{3} = 1.732$)

Sol)



Let AB be the lighthouse. Let C and D be the position of the two ships.

Then, $AB = 200\,m$

$\angle ACB = 30°$, $\angle ADB = 45°$

In the right angled ΔBAC, $\tan 30° = \frac{AB}{AC}$

$$\frac{1}{\sqrt{3}} = \frac{200}{AC} \Rightarrow AC = 200\sqrt{3} \longrightarrow (1)$$

In the right angled ΔBAD, $\tan 45° = \frac{AB}{AD}$

$$1 = \frac{200}{AD} \Rightarrow AD = 200 \longrightarrow (2)$$

$CD = AC + AD$

$= 200\sqrt{3} + 200$ (From (1) & (2))

$CD = 200(\sqrt{3}+1)$

$CD = 200(1.732+1)$

$CD = 200(2.732)$

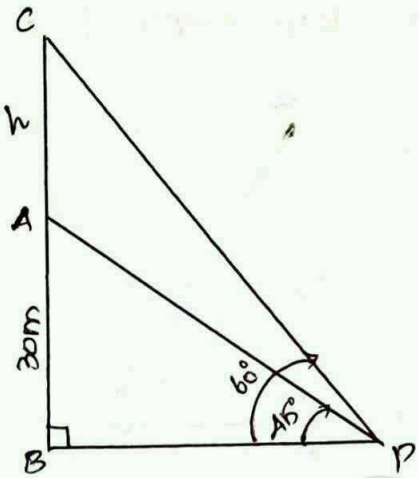
$CD = 546.4$

Distance between two ships is 546.4m.

Example 6.22

From a point on the ground, the angle of elevation of the bottom and top of a tower fixed at the top of a 30m high building are 45° and 60° respectively. Find the height of the tower. ($\sqrt{3} = 1.732$)

Sol



Let AC be the height of the tower.

Let AB be the height of the building

Then, AC = h metres , AB = 30m

In the right angled ΔCBP, $\angle CPB = 60°$

$$\tan\theta = \frac{BC}{BP}$$

$$\tan 60° = \frac{AB+AC}{BP}$$

$$\sqrt{3} = \frac{30+h}{BP} \longrightarrow (1)$$

In the right angled ΔABP, $\angle APB = 45°$

$$\tan\theta = \frac{AB}{BP}$$

$$\tan 45° = \frac{30}{BP}$$

$$1 = \frac{30}{BP} \Rightarrow BP = 30 \longrightarrow (2)$$

substitute (2) in (1)

$$\sqrt{3} = \frac{30+h}{30}$$

$$30 + h = 30\sqrt{3}$$

$$h = 30\sqrt{3} - 30$$

$$h = 30(\sqrt{3} - 1)$$

$$h = 30(1.732 - 1)$$

$$h = 30(0.732)$$

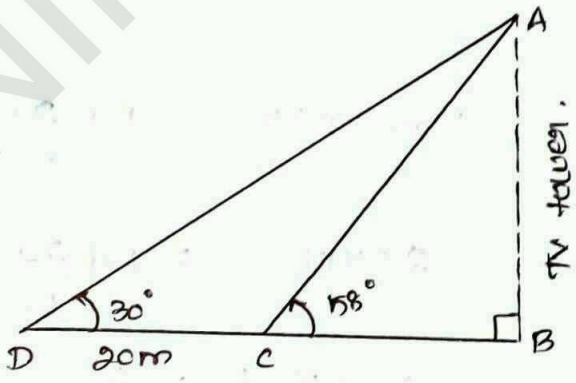
$$h = 21.96$$

Hence, the height of the tower is 21.96m.

Example 6.23

A TV tower stands vertically on a bank of a canal. the tower is watched from a point on the other bank directly opposite to it. The angle of elevation of the top of the tower is 58°. From another point 20m away from this point on this line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal. ($\tan 58° = 1.6003$).

Sol



Let AB be the height of the TV tower.

$CD = 20m$

Let BC be the width of the canal

In the right angled ΔABC, $\tan 58° = \frac{AB}{BC}$

$$1.5003 = \frac{AB}{BC} \longrightarrow (1)$$

In the right angled ΔABD, $\tan 30^\circ = \frac{AB}{BD}$

$$\tan 30^\circ = \frac{AB}{BC+CD}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BC+20} \longrightarrow (2)$$

dividing (1) by (2)

$$\frac{1.6003}{\frac{1}{\sqrt{3}}} = \frac{AB/BC}{\frac{AB}{BC+20}}$$

$$\frac{1.6003}{\frac{1}{\sqrt{3}}} = \frac{AB}{BC} \times \frac{BC+20}{AB}$$

$$\frac{1.6003}{\frac{1}{\sqrt{3}}} = \frac{BC+20}{BC}$$

$$1.6003 \times \frac{\sqrt{3}}{1} = \frac{BC}{BC} + \frac{20}{BC}$$

$$1.6003 \times 1.732 = 1 + \frac{20}{BC}$$

$$2.7717 = 1 + \frac{20}{BC}$$

$$2.7717 - 1 = \frac{20}{BC}$$

$$1.7717 = \frac{20}{BC}$$

$$BC = \frac{20}{1.7717} = 11.29 \longrightarrow (3)$$

In (1) substitute (3)

$$1.6003 = \frac{AB}{11.29}$$

$$AB = 1.6003 \times 11.29.$$

$AB = 18.07$

Hence, the height of the tower is ~~17.49~~ 18.07 m and the width of the canal is 11.29 m.

Example 6.24

An aeroplane sets off from G on a bearing of 24° towards H, a points 250 km away. At H it changes course and heads towards J deviates further by 55° and a distance of 180 km away.

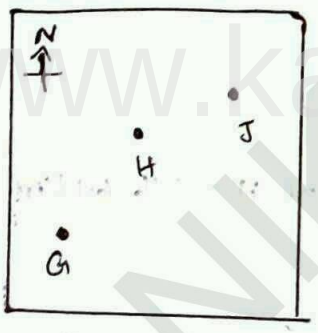
(i) How far is H to the North of G?

(ii) How far is H to the East of G?

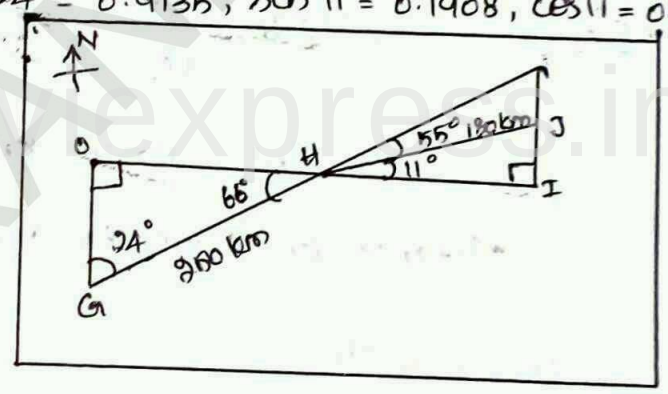
(iii) How far is J to the North of H?

(iv) How far is J to the East of H?

[$\sin 24° = 0.4067$, $\cos 24° = 0.9135$, $\sin 11° = 0.1908$, $\cos 11° = 0.9816$]



(a)



(b)

Sol

(i) In the right angled ΔGOH, $\cos 24° = \frac{OG}{GH}$

$$0.9135 = \frac{OG}{250} \; ; \quad OG = 0.9135 \times 250$$

$$OG = 228.38 \text{ km}$$

Distance of H to the north of G = 228.38 km.

(ii) In the right angled ΔGOH,

$$\sin 24° = \frac{OH}{GH}$$

$$0.4067 = \frac{OH}{250} \; ; \quad OH = 0.4067 \times 250$$

$OH = 101.68$

Distance of H to the East of G = 101.68 km

(iii) In the right angled ΔHIJ,

$$\sin H^\circ = \frac{IJ}{HJ}$$

$$0.1908 = \frac{IJ}{180} \; ; \; IJ = 0.1908 \times 180$$

$$IJ = 34.34 \text{ km}$$

Distance of J to the North of H = 34.34 km

(iv) In the right angled ΔHIJ,

$$\cos 11^\circ = \frac{HI}{HJ}$$

$$0.9816 = \frac{HI}{180} \; ; \; HI = 0.9816 \times 180$$

$$HI = 176.69 \text{ km}$$

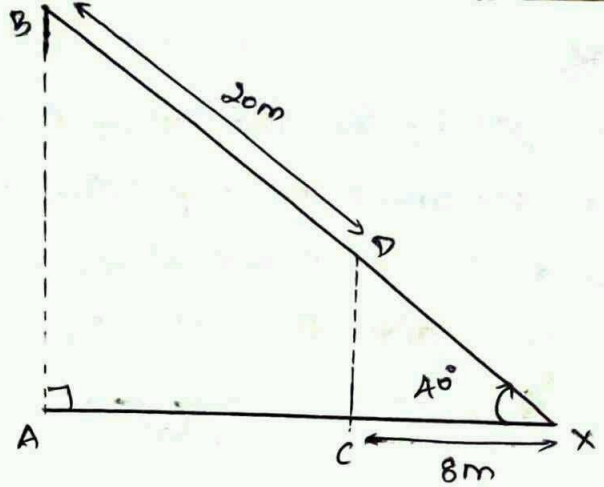
Distance of J to the East of H = 176.69 km.

Example 6.25:

Two trees are standing on flat ground. the angle of elevation of the top of the both the trees from a point x on the ground is 40°. If the horizontal distance between x and the smaller tree is 8m and the distance of the top of the two trees is 20m, calculate

(i) The distance between the point x and the top of the smaller tree.

(ii) The horizontal distance between the two trees. ($\cos 40^\circ = 0.7660$).



Sol

Let AB be the height of the bigger tree and CD be the height of the smaller tree. and X is the point on the ground.

(i) In the right angled, ΔXCD, $\cos 40° = \frac{CX}{XD}$

$$XD = \frac{8}{0.7660} = 10.44m$$

$\therefore$ The distance between X and top of the smaller tree $= XD = 10.44m$.

(ii) In the right angle ΔXAB,

$$\cos 40° = \frac{AX}{BX} = \frac{AC + CX}{BD + DX}$$

$$0.7660 = \frac{AC + 8}{20 + 10.44} = \frac{AC + 8}{30.44}$$

$$AC + 8 = 0.7660 \times 30.44$$

$$AC + 8 = 23.32$$

$$AC = 23.32 - 8$$

$$AC = 15.32$$

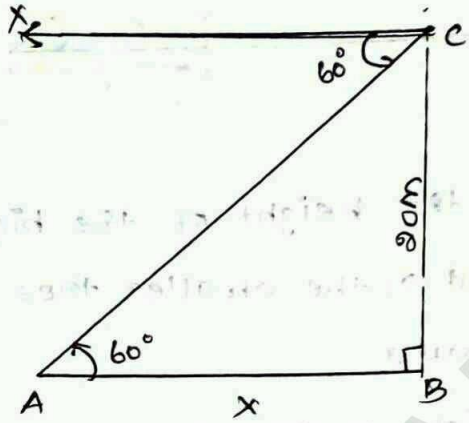
$\therefore$ The horizontal distance between two trees $=$

$AC = 15.32m$

Example 6.26

A player sitting on the top of a tower of height 20m observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball. ($\sqrt{3} = 1.732$).

Sol



Let BC be the height of the tower and A be the position of the ball lying on the ground.

Then, BC = 20m and $\angle XCA = \angle CAB = 60°$

Let AB = x metres.

In the right angled ΔABC,

$$\tan 60° = \frac{BC}{AB}$$

$$\sqrt{3} = \frac{20}{x}$$

$$x = \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{20\sqrt{3}}{3}$$

$$x = \frac{20 \times 1.732}{3}$$

$$x = 11.55m.$$

Hence, the distance between the foot of the tower and the ball is 11.55m.

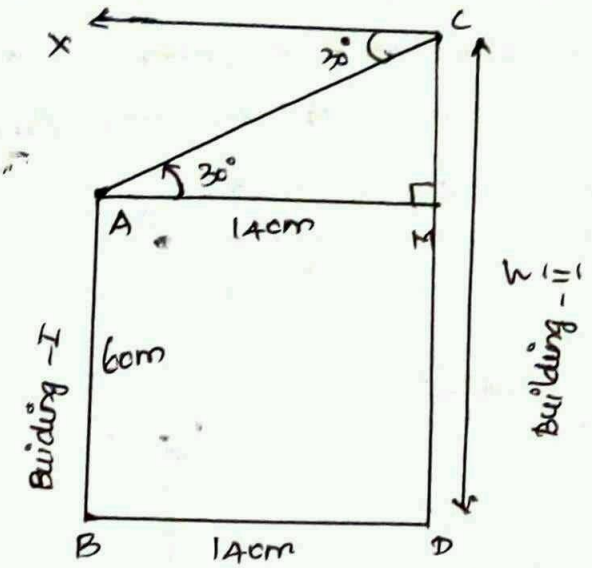
Example 6.27

The horizontal distance between two buildings is 140m. The angle of depression of the top of the first building when seen from the top of the second building is 30°.

If the height of the first building is 60m, find the height of the second building. ($\sqrt{3} = 1.732$).

(187)

Sol



The height of the first building AB = 60m,

AB = MD = 60m

Let the height of the second building CD = h

distance BD = 140m

~~AB~~ AM = BD = 140m

From the figure,

$\angle XCA = 30° = \angle CAM$

In the right angled $\triangle$ AMC,

$$\tan 30° = \frac{CM}{AM}$$

$$\frac{1}{\sqrt{3}} = \frac{CM}{140}$$

$$CM = \frac{140}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{140\sqrt{3}}{3}$$

$$CM = \frac{140 \times 1.732}{3}$$

$$CM = 80.83$$

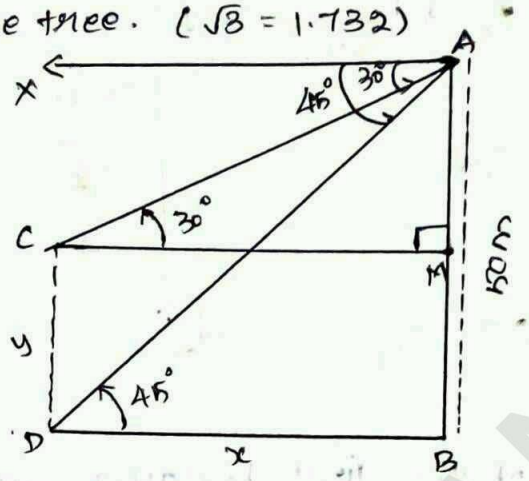
Now, $h = CD = CM + MD = 80.83 + 60 = 140.83$

$\therefore$ the height of the second building is 140.83m.

Example 6.28

From the top of the tower 50 m high, the angles of depression of the top and bottom of a tree are observed to be 30° and 45° respectively. Find the height of the tree. ($\sqrt{3} = 1.732$)

Sol



The height of the tower AB = 50m

Let the height of the tree CD = y and BD = x

From the diagram,

$\angle XAC = 30° = \angle ACM$ and $\angle XAD = 45° = \angle ADB$

In the right angled ΔABD,

$$\tan 45° = \frac{AB}{BD}$$

$$1 = \frac{50}{x}$$

$$x = 50m$$

In the right angled ΔAMC

$$\tan 30° = \frac{AM}{CM}$$

$$\frac{1}{\sqrt{3}} = \frac{AM}{50} \quad [\because DB = CM]$$

$$AM = \frac{50}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{50\sqrt{3}}{3} = \frac{50 \times 1.732}{3}$$

$$AM = 28.87m$$

$\therefore$ Height of the tree = CD = MB = AB − AM

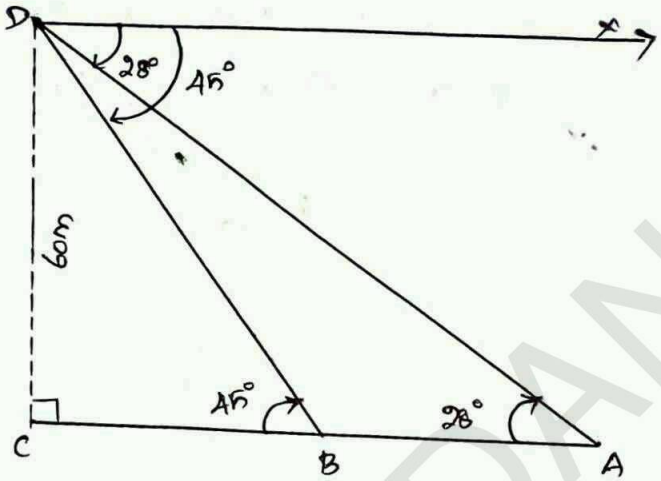
$= 50 - 28.87$

$= 21.13m.$

Example 6.29

As observe from the top of a 60m high lighthouse from the sea level, the angles of depression of two ships are 28° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships. (tan 28° = 0.5317)

Sol



Let the observer on the lighthouse CD be at D.

Height of the lighthouse CD = 60m

From the diagram,

$\angle XDA = 28° = \angle DAC$ and

$\angle XDB = 45° = \angle DBC$

In the right angled ΔDCB, $\tan 45° = \frac{DC}{BC}$

$1 = \frac{60}{BC}$

$BC = 60m$

In the right angled ΔDCA, $\tan 28° = \frac{DC}{AC}$

$0.5317 = \frac{60}{AC}$

$AC = \frac{60}{0.5317} = 112.85$

Distance between two ships $AB = AC - BC$

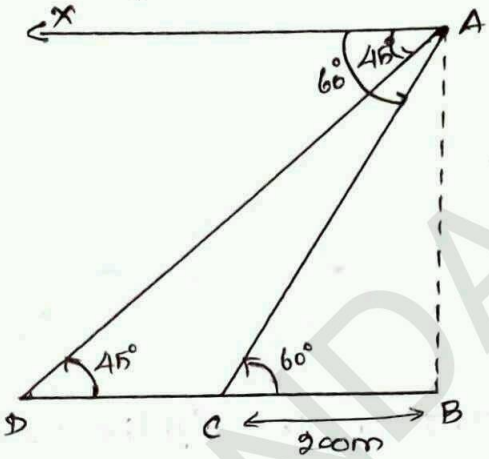
$= 112.85 - 60$

$= 52.85m$

Example 6.30

A man is watching a boat speeding away from the top of the tower. the boat makes the angle of depression of 60° with the man's eye when at a distance of 200m from the tower. After 10 seconds, the angle of depression becomes 45°. What is the approximate speed of the boat (in km/hr), assuming that it is sailing in still water? ($\sqrt{3} = 1.732$).

Sol.



Let AB be the tower.

Let C and D be the position of the boat

From the diagram,

$\angle XAC = 60° = \angle ACB$ and

$\angle XAD = 45° = \angle ADB$, $BC = 200m$

In the right angled ΔABC, $\tan 60° = \frac{AB}{BC}$

$$\sqrt{3} = \frac{AB}{200}$$

$$AB = 200\sqrt{3} \rightarrow (1)$$

In the right angled ΔABD, $\tan 45° = \frac{AB}{BD}$

$$1 = \frac{200\sqrt{3}}{BD} \quad (\text{from } (1))$$

$$BD = 200\sqrt{3}$$

$$CD = BD - BC$$

$$CD = 200\sqrt{3} - 200$$

$= 200(\sqrt{3}-1)$

$= 200(1.732-1)$

$= 200(0.732)$

$= 146.4$

It is given that the distance CD is covered in 10 seconds that is, the distance 146.4 m covered in 10 seconds

$\therefore$ Speed of the boat $= \dfrac{\text{distance}}{\text{time}}$

$= \dfrac{146.4}{10} = 14.64\ m/s$

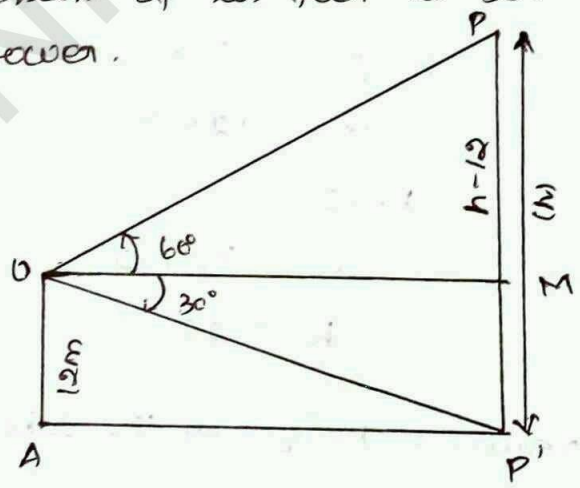
$\Rightarrow 14.64 \times \dfrac{3600}{1000}$ km/hr

Speed of the boat $= 52.704$ km/hr.

Example 6.31

From the top of the 12 m high buildings, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 30°. Determine the height of the tower.

Sol



OA is the building. O is the point of observation of the top of the building OA. then OA = 12m

PP' is the cable tower with P as the top and P' as the bottom.

Then, the angle of elevation of P, $\angle MOP = 60°$

and, the Angle of depression of P', $\angle MOP' = 30°$.

Suppose, the height of the cable tower $PP' = h$ meters

through, O, draw $OM \perp PP'$ (92)

$$MP = PP' - MP' = h - OA = h - 12$$

In the right angled ΔOMP, $\frac{MP}{OM} = \tan 60°$

$$\frac{h-12}{OM} = \sqrt{3}$$

$$OM = \frac{h-12}{\sqrt{3}} \rightarrow (1)$$

In the right angled $\Delta OMP'$, $\frac{MP'}{OM} = \tan 30°$

$$\frac{12}{OM} = \frac{1}{\sqrt{3}}$$

$$OM = 12\sqrt{3} \rightarrow (2)$$

From (1) & (2)

$$\frac{h-12}{\sqrt{3}} = 12\sqrt{3}$$

$$h - 12 = 12\sqrt{3} \times \sqrt{3}$$

$$h - 12 = 12 \times 3$$

$$h - 12 = 36$$

$$h = 36 + 12$$

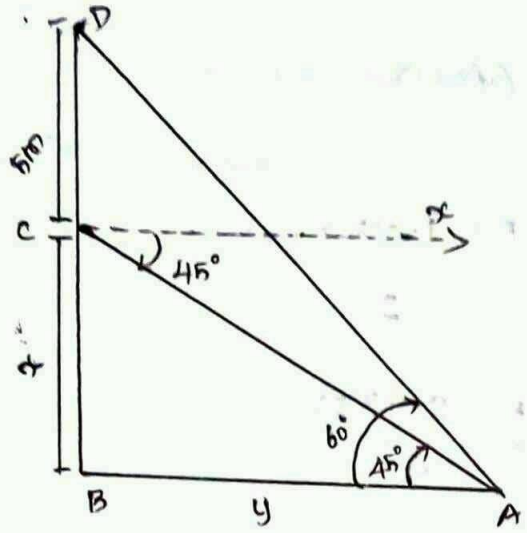
$$h = 48$$

$\therefore$ The required height of the cable tower is 48m.

Example 6.32

A pole 5m high is fixed on the top of a tower. the angle of elevation of the top of the pole observed from a point 'A' on the ground is 60° and the angle of depression to the point 'A' from the top of the tower is 45°. Find the height of the tower. ($\sqrt{3} = 1.732$).

Sol



Let BC be the height of the tower and CD be the height of the pole.

Let 'A' be the point of observation

Let $BC = x$ and $AB = y$

From the diagram,

$\angle BAD = 60^\circ$ and $\angle XCA = 45^\circ = \angle BAC$

In the right angled ΔABC, $\tan 45^\circ = \frac{BC}{AB}$

$$1 = \frac{x}{y}$$

$$x = y \longrightarrow (1)$$

In the right angled ΔABD, $\tan 60^\circ = \frac{BD}{AB} = \frac{BC + CD}{AB}$

$$\sqrt{3} = \frac{x+5}{y}$$

$$\sqrt{3}y = x+5 \longrightarrow (2)$$

substitute (1) in (2)

$$\sqrt{3}y = x+5$$

$$\sqrt{3}x = x+5$$

$$5 = \sqrt{3}x - x$$

$$5 = (\sqrt{3}-1)x$$

$$x = \frac{5}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{5(\sqrt{3}+1)}{3-1}$$

$$= \frac{5(1.732+1)}{2}$$

$$= \frac{5(2.732)}{2}$$

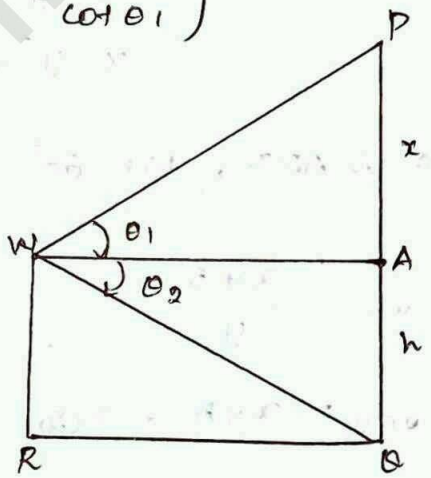
$$= \frac{13.66}{2} = 6.83$$

Hence, height of the tower is 6.83m.

Example 6.33

From a window (h metres high above the ground) of a house in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are θ_1 and θ_2 respectively. Show that the height of the opposite house is $h\left(1+\frac{\cot\theta_2}{\cot\theta_1}\right)$

Sol



Let W be the point on the window where the angles of elevation and depression are measured. Let PQ be the house on the opposite side.

Then, WA is the width of the street

Height of the window = h metres

$= AQ \ (WR = AQ)$

Let $PA = x$ meters

In the right angled ΔPAW, $\tan\theta_1 = \frac{AP}{AW}$

$$\tan\theta_1 = \frac{x}{AW}$$

$$AW = \frac{x}{\tan\theta_1}$$

$$AW = x\cot\theta_1 \rightarrow (1)$$

In the right angled ΔQAW, $\tan\theta_2 = \frac{AQ}{AW}$

$$\tan\theta_2 = \frac{h}{AW}$$

$$AW = \frac{h}{\tan\theta_2}$$

$$AW = h\cot\theta_2 \rightarrow (2)$$

From (1) & (2)

$$x\cot\theta_1 = h\cot\theta_2$$

$$x = h\frac{\cot\theta_2}{\cot\theta_1}$$

$\therefore$ Height of the opposite house $= PA + AQ = x + h$

$$h\left(\frac{\cot\theta_2}{\cot\theta_1}\right) + h$$

$$= h\left[1 + \frac{\cot\theta_2}{\cot\theta_1}\right]$$

Hence proved.

Xth HAND WRITTEN MATERIALS

CHAPTER – 7 (Examples)

MENSURATION

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P.G Teacher Mathematics

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CHAPTER - 7

MENSURATION

Example 7.1

A cylindrical drum has a height of 20cm and base radius of 14cm. Find its curve surface Area and the total surface Area.

Sol

Given;

Height of the cylinder $h = 20cm$

radius $r = 14cm$.

Now,

C.S.A of the cylinder $= 2\pi rh$ sq. units

$= 2 \times \frac{22}{7} \times 14 \times 20$

$= 2 \times 22 \times 2 \times 20$

C.S.A of the cylinder $= 1760\ cm^2$

T.S.A of the cylinder $= 2\pi r(h+r)$ sq. units.

$= 2 \times \frac{22}{7} \times 14\ (20+14)$

$= 2 \times 22 \times 2 \times 34$

T.S.A of the cylinder $= 2992\ cm^2$

$\therefore$ C.S.A $= 1760\ cm^2$, T.S.A $= 2992\ cm^2$

Example 7.2

The curved surface area of a right circular cylinder of height 14cm is $88\ cm^2$. Find the diameter of the cylinder.

Sol

Given:

$h = 14cm$

C.S.A of the cylinder = 88 sq.cm

$2\pi rh = 88$

$2 \times \frac{22}{7} \times r \times 14 = 88$

$2 \times 22 \times 2 \times r = 88$

$2r = \frac{88}{44}$

$2r = 2$

$\therefore$ diameter $2r = 2$ cm.

Example 7.3

A garden roller whose length is 3m long and whose diameter is 2.8m is rolled to level a garden. How much area will it cover in 8 revolutions?

Sol) Given:

diameter $d = 2.8$m ; height = 3m

$2r = 2.8$m

$r = \frac{2.8}{2} = 1.4$m

Area covered in one revolution = curve surface area of the cylinder

$= 2\pi rh$ sq. units

$= 2 \times \frac{22}{7} \times 1.4 \times 3$

$= 26.4$

Area covered in 1 revolution = $26.4 m^2$

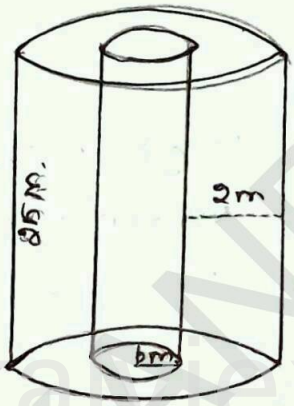
Area covered in 8 revolution $= 8 \times 26.4$

$= 211.2$

$\therefore$ Area covered is $211.2\ m^2$.

Example 7.4

If one litre of paint covers $10\ m^2$, how many litres of paint is required to paint the internal and external surface area of a cylindrical tunnel whose thickness is 2m, internal radius is 6m and height is 25m.



Sol) Given:

height $h = 25m$, thickness $= 2m$

internal radius $r = 6m$

External radius $R = r +$ thickness

$R = 6 + 2$

$R = 8\ m$

C.S.A of the cylindrical tunnel = C.S.A of the hallow cylinder

C.S.A of the hallow cylinder $= 2\pi (R+r)h$ sq.units

$= 2 \times \frac{22}{7}(8+6) \times 25$

C.S.A of the cylindrical tunnel $= 2200\ m^2$

Area covered by one litre of paint $= 10\ m^2$

Number of litres required to paint the tunnel

$$= \frac{2200}{10} = 220$$

$\therefore$ 220 Litres of paint is needed to paint the tunnel.

Example 7.5

The Radius of a concial tent is 7m and the height is 24m. calculate the length of the canvas used to make the tent if the width of the rectangular canvas is 4m ?

Sol

Let r and h be the radius and height of the cone respectively.

radius $r = 7m$ and height $h = 24m$

Hence, $l = \sqrt{r^2 + h^2}$

$$l = \sqrt{7^2 + 24^2}$$

$$= \sqrt{49 + 576}$$

$$l = \sqrt{625} = 25m$$

C.S.A of the conical tent $= \pi r l$ sq. units

Area of the canvas $= \frac{22}{7} \times 7 \times 25 = 550m^2$

length of the canvas $= \frac{\text{Area of the canvas}}{\text{width}}$

$$= \frac{550}{4}$$

$$= 137.5m$$

$\therefore$ The length of the canvas is 137.5m

Example 7.6

If the total surface area of a cone of radius 7cm is 704 cm², then find its slant height.

Sol

Given:

radius $r = 7cm$

total surface area of the cone $= \pi r(l+r)$ sq. unit

$$T.S.A = 704 \text{ cm}^2$$

$$704 = \frac{22}{7} \times 7(l+7)$$

$$704 = 22(l+7)$$

$$l+7 = \frac{704}{22}$$

$$l+7 = 32$$

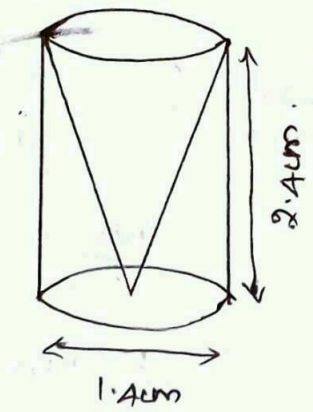
$$l = 32-7$$

$$l = 25cm$$

∴ Slant height of the cone is 25cm.

Example 7.7

From a solid cylinder whose height is 2.4cm and diameter 1.4cm, a conical cavity of the same height and base is hollowed out. Find the total surface area of the remaining solid.



Sol

Let h and r be the height and radius of the cone and cylinder.

Let l be the slant height of the cone.

Given: $h = 2.4\,cm$, $d = 1.4\,cm$

$r = 0.7\,cm$

Total surface area of the remaining solid $\Big\}$ = C.S.A of the cylinder + C.S.A of the cone + area of the bottom

$= 2\pi rh + \pi rl + \pi r^2$ sq. units

$= \pi r(2h + l + r)$ sq. units.

$l = \sqrt{r^2 + h^2} = \sqrt{(0.7)^2 + (2.4)^2} = \sqrt{0.49 + 5.76}$

$= \sqrt{6.25}$

$l = 2.5\,cm$

Area of the remaining solid $= \pi r(2h + l + r)$ sq. units

$= \frac{22}{7} \times 0.7[(2 \times 2.4) + 2.5 + 0.7]$

$= \frac{22}{7} \times 0.7(8)$

$= \frac{22}{7} \times 5.6$

$= 3.14 \times 5.6$

$= 17.6$

$\therefore$ Total surface Area of the remaining solid is $17.6\,m^2$.

Example 7.8

Find the diameter of a sphere whose surface area is $154\,cm^2$

Sol

Let r be the radius of the sphere.

Surface area of sphere $= 154\ m^2$

$$4\pi r^2 = 154$$

$$4 \times \frac{22}{7} \times r^2 = 154$$

$$r^2 = 154 \times \frac{1}{4} \times \frac{7}{22}$$

$$r^2 = \frac{49}{4}$$

$$r = \frac{7}{2}$$

$\therefore$ diameter $2r = 7m$.

Example 7.9

The radius of the spherical balloon increases from 12 cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases.

Sol

Let r_1 and r_2 be the radii of the balloons.

Given: $\frac{r_1}{r_2} = \frac{12}{16} = \frac{3}{4}$

Ratio of C.S.A of balloons $= \frac{4\pi r_1^2}{4\pi r_2^2}$

$$= \frac{r_1^2}{r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

$$= \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

$\therefore$ Ratio of C.S.A of balloon is 9 : 16.

Example 7.10

If the base area of a hemispherical solid is 1386 sq.m then find its total surface Area ?

Sol

Let r be the radius of the hemisphere

$$\text{Base Area} = \pi r^2 = 1386 \text{ sq.m}$$

$$T.S.A = 3\pi r^2 \text{ sq.m}$$

$$= 3 \times 1386$$

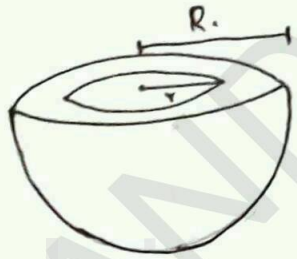
$$= 4158$$

$\therefore$ T.S.A of the hemispherical solid is $4158 m^2$.

Example 7.11

The internal and external radii of a hollow hemispherical shell are 3m and 5m respectively. Find the T.S.A and C.S.A of the shell.

Sol



Let the internal and external radii of the hemisphere shell be r and R respectively.

$$R = 5m, \quad r = 3m$$

$$C.S.A \text{ of the shell} = 2\pi (R^2 + r^2) \text{ sq. units}$$

$$= 2 \times \frac{22}{7} \times (25 + 9)$$

$$= 213.71$$

$$T.S.A \text{ of the shell} = \pi (3R^2 + r^2) \text{ sq. units}$$

$$= \frac{22}{7} (3 \times (5)^2 + 3^2)$$

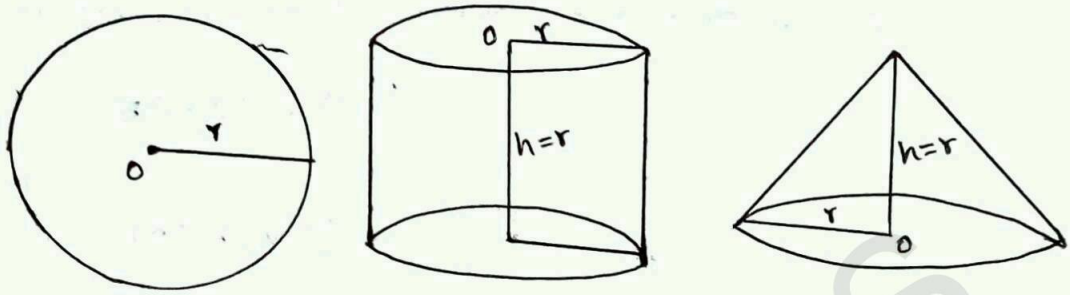
$$= \frac{22}{7} (3 \times 25 + 9)$$

$$= \frac{22}{7} (75 + 9) \Rightarrow 264$$

$\therefore$ C.S.A $= 213.71 m^2$ and T.S.A $= 264 m^2$.

Example 7.12

A sphere, a cylinder and a cone are of the same radius, where as cone and cylinder are of same height. Find the ratio of their curved surfaced Area.



Sol

Required Ratio = C.S.A of the sphere : C.S.A of the cylinder : C.S.A of the cone

$= 4\pi r^2 : 2\pi r h : \pi r l$

$[l = \sqrt{r^2+h^2} = \sqrt{2r^2}$

$= \sqrt{2}r]$

$= 4\pi r^2 : 2\pi r h : \pi r \sqrt{2}r$

$= 4\pi r^2 : 2\pi r^2 : \sqrt{2}\pi r^2$

$= 4 : 2 : \sqrt{2}$

$= \frac{4\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} : \frac{2}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} : \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{4\times\sqrt{2}}{2} : \frac{2\sqrt{2}}{2} : 1$

$= 2\sqrt{2} : \sqrt{2} : 1$

Example 7.13

The slant height of a frustum of a cone is 5m and the radii of its ends are 4cm and 1cm. Find its curved surface Area.

Sol Let l, R, and r be the slant height, top radius and bottom radius of the frustum.

$l = 5\,cm$, $R = 4\,cm$, $r = 1\,cm$

C.S.A of the frustum $= \pi (R+r) l$ sq. units

$= \frac{22}{7} \times (4+1)5$

$= \frac{22}{7} \times (5)(5)$

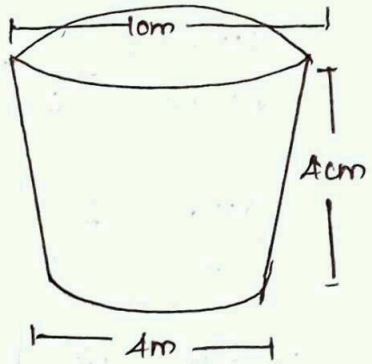
$= \frac{22}{7} \times 25$

$= \frac{550}{7}$

$\therefore$ C.S.A $= 78.57\,cm^2$.

Example 7.14

An industrial metallic bucket is in the shape of the frustum of a right circular cone whose top and bottom diameters are 10m and 4m and whose height is 4m. Find the curved and total surface area of the bucket.



Sol Let h, l, R and r be the height, slant height, outer radius and inner radius of the frustum.

Given: diameter of the top $= 10\,m$

Radius of the top $R = 5\,m$.

diameter of the bottom $= 4\,m$

Radius of the bottom $r = 2m$

height $h = 4m$

$$l = \sqrt{h^2 + (R^2 - r^2)}$$

$$= \sqrt{4^2 + (5-2)^2}$$

$$l = \sqrt{4^2 + 3^2}$$

$$l = \sqrt{16+9}$$

$$l = \sqrt{25} = 5m$$

$$C.S.A = \pi (R+r) l \text{ sq. units.}$$

$$= \frac{22}{7}(5+2) \times 5$$

$$= \frac{22}{7} \times 7 \times 5 = 110m^2$$

$$T.S.A = \pi (R+r) l + \pi R^2 + \pi r^2 \text{ sq. units}$$

$$= \frac{22}{7}\left[(5+2)5 + 5^2 + 2^2\right]$$

$$= \frac{22}{7}\left[35 + 25 + 4\right]$$

$$= \frac{1408}{7} = 201.14\ m^2$$

$\therefore$ $C.S.A = 110m^2$, $T.S.A = 201.14\ m^2$

Example 7.15

Find the volume of the cylinder whose height is 2m and whose base Area is $250m^2$

Sol Let r and h be the radius and height of the cylinder respectively.

Given: height $h = 2m$, base Area $= 250\ m^2$

volume of a cylinder $= \pi r^2 h$ cu. units

$= $ base Area $\times h$

$= 250 \times 2$

$= 500\ m^2$

$\therefore$ volume of the cylinder $= 500\ m^2$

Example 7.16

The volume of a cylindrical water tank is 1.078×10^6 litres. If the diameter of the tank is 7m, find its height.

Sol

Let r and h be the radius and height of the cylinder respectively.

volume of the tank $= 1.078 \times 10^6 = 1078000$ litre

$= 1078\ m^3$ $\left[\because 1l = \frac{1}{1000} m^3\right]$

diameter $= 7m \Rightarrow$ radius $r = \frac{7}{2} m$

volume of the tank $= \pi r^2 h$ cu. units

$$1078 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h$$

$$h = 1078 \times \frac{2}{77}$$

$$h = 28m$$

$\therefore$ Height of the tank is 28m.

Example 7.17

Find the volume of the iron used to make a hollow cylinder of height 9cm and whose internal and external radii are 21 cm and 28cm respectively.

Sol

Let r, R and h be the internal radius, external radius and height of the hollow cylinder respectively.

$r = 21\ cm,\ R = 28cm,\ h = 9cm$

$\therefore$ volume of hollow cylinder $= \pi(R^2 - r^2)h$ cu. units

$$= \frac{22}{7}(28^2 - 21^2) \times 9$$

$$= \frac{22}{7} \times (784 - 441) \times 9$$

$$= 9702$$

$\therefore$ volume of iron used $= 9702\ cm^3$

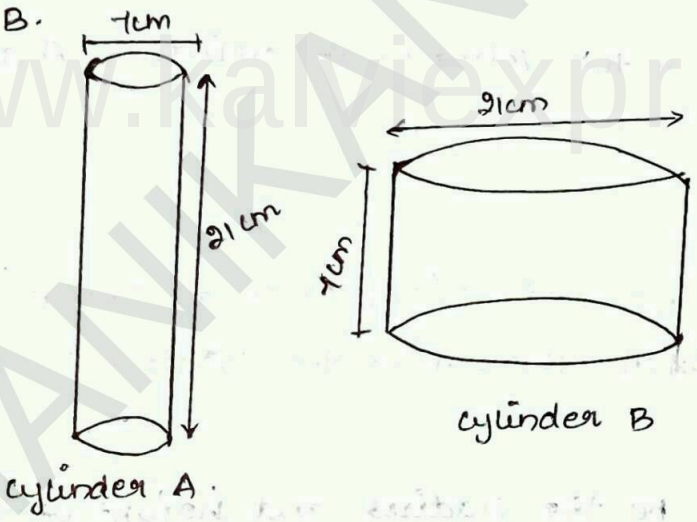
Example 7.18

For the cylinders A and B.

(i) find out the cylinder whose volume is greater.

(ii) verify whether the cylinder with greater volume has greater total surface Area.

(iii) Find the ratios of the volumes of the cylinders A and B.



Sol (i) volume of cylinder $= \pi r^2 h$ cu. units

volume of cylinder A $= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 21$

$$= 808.5\ cm^3$$

volume of cylinder B $= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 7$

$$= 2425.5\ cm^3$$

$\therefore$ volume of cylinder B is greater than volume of cylinder A.

(ii) T.S.A of cylinder $= 2\pi r(h+r)$ sq. units

T.S.A of cylinder A $= 2 \times \frac{22}{7} \times \frac{7}{2} \times (21+3.5)$

$= 22 \times (24.5)$

$= 539 \text{ cm}^2$

T.S.A of cylinder B $= 2 \times \frac{22}{7} \times \frac{21}{2} \times (7+10.5)$

$= 66 \times 17.5$

$= 1155 \text{ cm}^2$

Hence verified that cylinder B with greater volume has a greater surface Area.

(iii) $\frac{\text{Volume of cylinder A}}{\text{Volume of cylinder B}} = \frac{808.5}{2425.5} = \frac{1}{3}$

$\therefore$ Ratio of the volumes of cylinders A and B is 1:3.

Example 7.19

The volume of a solid right circular cone is 11088 cm^3. If its height is 24 cm, then find the radius of the cone.

Sol Let r and h be the radius and height of the cone respectively.

Volume of the cone $= 11088 \text{ cm}^3$

$\frac{1}{3}\pi r^2 h = 11088$

$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 11088$

$r^2 = 11088 \times \frac{1}{24} \times 3 \times \frac{7}{22}$

$r^2 = 441$

$\therefore$ radius of the cone $r = 21$ cm.

Example 7.20

The ratio of the volume of two cones is 2:3. Find the ratio of their radii if the height of second cone is double the height of the first. (210)

Sol

Let r_1 and h_1 be the radius and height of the cone - I

Let r_2 and h_2 be the radius and height of the cone - II

$h_2 = 2h_1$ and $\dfrac{\text{Volume of the cone I}}{\text{Volume of the cone II}} = \dfrac{2}{3}$

$$\frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{2}{3}$$

$$\frac{r_1^2}{r_2^2} \times \frac{h_1}{2h_1} = \frac{2}{3}$$

$$\frac{r_1^2}{r_2^2} = \frac{4}{3}$$

$$\frac{r_1}{r_2} = \frac{2}{\sqrt{3}}$$

$\therefore$ Ratio of their radii $= 2 : \sqrt{3}$

Example 7.21

The volume of a solid hemisphere is 29106 cm^3. Another hemisphere whose volume is two-third of the above is carved out. Find the radius of the new hemisphere

Sol Let r be the radius of the hemisphere

Volume of the hemisphere $= 29106\ cm^3$

Volume of new hemisphere $= \frac{2}{3}$ (volume of original sphere).

$= \frac{2}{3} \times 29106$

Volume of new hemisphere $= 19404 cm^3$

$$\frac{2}{3}\pi r^3 = 19404$$

$$r^3 = \frac{19404 \times 3 \times 7}{2 \times 22}$$

$$r^3 = 9261$$

$$r = \sqrt[3]{9261}$$

$$r = 21cm.$$

Example 7.22

Calculate the weight of a hollow brass sphere if the inner diameter is 14cm and thickness is 1mm, and whose density is $17.3\ g/cm^3$.

Sol

Let r and R be the inner and outer radii of the hollow sphere.

inner diameter $d = 14cm$

inner radius $r = 7cm$

thickness $= 1mm = \frac{1}{10} cm$

Outer Radius $R = 7 + \frac{1}{10} = \frac{71}{10} = 7.1cm$

Volume of hollow sphere $= \frac{4}{3}\pi (R^3 - r^3)$ cu.units

$$= \frac{4}{3} \times \frac{22}{7} ((7.1)^3 - (7)^3)$$

$$= \frac{4}{3} \times \frac{22}{7} (357.91 - 343)$$

$$= 62.48 cm^3$$

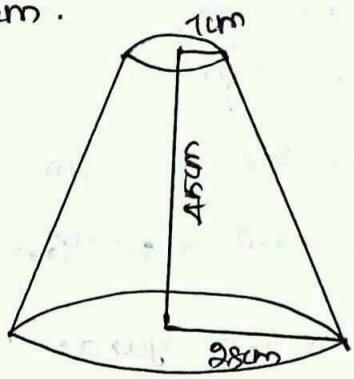
weight of brass in $1cm^3 = 17.3gm$

Total weight $= 17.3 \times 62.48 = 1080.90 gm$

$\therefore$ Total weight is 1080.90 grams.

Example 7.23

If the radii of the circular ends of a frustum which is 45cm high are 28cm and 7cm, find the volume of the frustum.



Sol

Let h, r, R be the height, top and bottom radii of the frustum.

$h = 45cm, \; R = 28cm, \; r = 7cm$

$$\text{volume} = \frac{1}{3}\pi[R^2 + Rr + r^2]h \text{ cu. units}$$

$$= \frac{1}{3} \times \frac{22}{7}[28^2 + (28 \times 7) + 7^2] \times 45$$

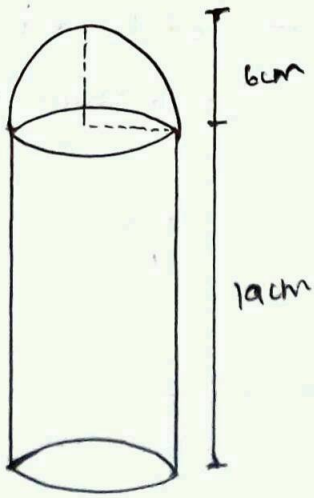
$$= \frac{1}{3} \times \frac{22}{7} \times 1029 \times 45$$

$$= 48510$$

$\therefore$ volume of the frustum is $48510 \, cm^3$.

Example 7.24

A toy is in the shape of a cylinder surmounted by a hemisphere. the height of the toy is 25cm. Find the total surface area of the toy if its common diameter is 12cm.



Sol

Let r and h be the radius and height of the cylinder respectively.

Given: diameter $d = 12cm$

radius $r = 6cm$.

Total height of the toy is 25cm

$\therefore$ Height of the cylindrical portion $= 25 - 6 = 19cm$.

T.S.A of the toy = C.S.A of the cylinder + C.S.A of the hemisphere + Base Area of the cylinder

$$= 2\pi rh + 2\pi r^2 + \pi r^2$$

$$= \pi r(2h + 2r + r)$$

$$= \pi r(2h + 3r) \text{ sq. units}$$

$$= \frac{22}{7} \times 6(2 \times 19 + 3 \times 6)$$

$$= \frac{22}{7} \times 6(38 + 18)$$

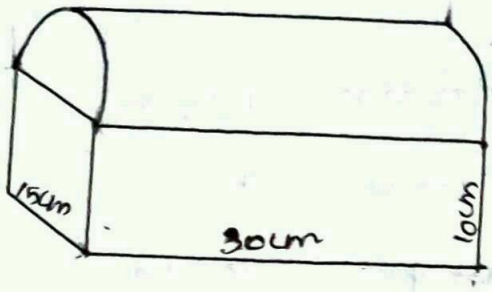
$$= \frac{22}{7} \times 6 \times 56$$

$$= 1056$$

$\therefore$ T.S.A of the toy is $1056\ cm^2$.

Example 7.25

A jewel box is in the shape of a cuboid of dimension 30cm × 15cm × 10cm surmounted by a half part of a cylinder as shown. Find the volume of the box



Sol

Let l, b, h be the length, breadth and height of the cuboid. Also let us take r and h_2 be the radius and height of the cylinder.

Volume of the box = volume of the cuboid + $\frac{1}{2}$ (volume of cylinder)

$$= (l \times b \times h_1) + \frac{1}{2}(\pi r^2 h_2) \text{ cu. units}$$

$$= (30 \times 15 \times 10) + \frac{1}{2}\left(\frac{22}{7} \times \frac{15}{2} \times \frac{15}{2} \times 20\right)$$

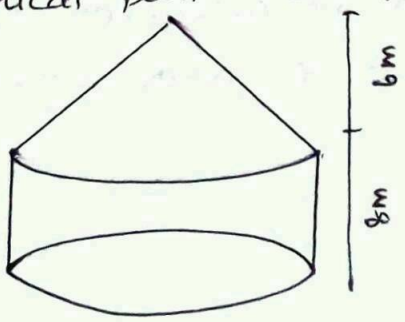
$$= 4500 + 2651.79$$

$$= 7151.79$$

$\therefore$ volume of the box $= 7151.79 \text{ cm}^3$.

Example 7.26

Arul has to make arrangements for the accommodation of 150 persons for his family function. For this purpose, he plans to build a tent which is in the shape of cylinder surmounted by a cone. Each person occupies 4 sq.m of the space on ground and 40 cu.metres of air to breathe. What should be the height of the conical part of the tent if the height of cylindrical part is 8 m?



Sol) Let h_1 and h_2 be the height of cylinder and cone respectively.

Area of one person = 4 sq.m

total number of person = 150

total base area = 150 × 4

$\pi r^2 = 600 \longrightarrow (1)$

volume of air required for 1 person = $40 m^3$

total volume of air required for 150 person = 150 × 40

$= 6000 m^3$

$$\pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2 = 6000$$

$$\pi r^2 \left(h_1 + \frac{1}{3} h_2 \right) = 6000$$

$$600 \left(8 + \frac{1}{3} h_2 \right) = 6000$$

$$8 + \frac{1}{3} h_2 = \frac{6000}{600}$$

$$\frac{1}{3} h_2 = 10 - 8 = 2$$

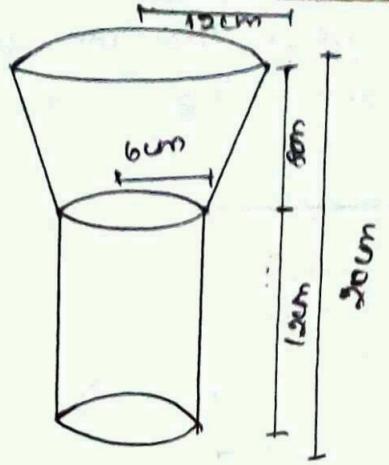
$$h_2 = 2 \times 3$$

$$h_2 = 6m$$

∴ Height of the conical tent h_2 is 6m

Example 7.27

A funnel consists of a frustum of a cone attached to a cylindrical portion 12cm long attached at the bottom. If the total height be 20cm, diameter of the cylindrical portion be 12cm and the diameter of the top of the funnel be 24cm. Find the outer surface area of the funnel.



Sol

Let R, r be the top and bottom radii of the frustum.

Let h_1, h_2 be the height of the frustum and cylinder respectively.

Given: $R = 12cm$, $r = 6cm$, $h_2 = 12cm$

$h_1 = 20 - 12 = 8cm$.

Slant height of the frustum $l = \sqrt{(R-r)^2 + h_1^2}$ units

$= \sqrt{36+64}$

$= \sqrt{100}$

$l = 10cm$

Outer surface Area $= 2\pi r h_2 + \pi (R+r) l$ sq.units

$= \pi [2rh_2 + (R+r)l]$

$= \pi [(2 \times 6 \times 12) + (18 \times 10)]$

$= \pi [144 + 180]$

$= \frac{22}{7} \times 324$

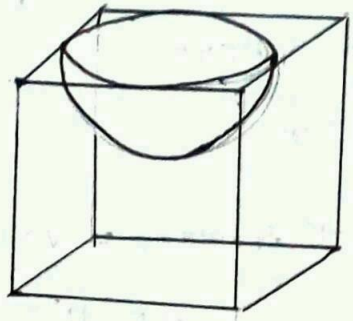
$= 1018.28$

$\therefore$ outer surface area of the funnel is $1018.28cm^2$.

Example 7.28

A hemispherical section is cut out from one face of a cubical block such that the diameter l of the hemisphere

is equal to side length of the cube. Determine the surface area of remaining solid. (2M)



Sol

Let r be the radius of the hemisphere

Given: diameter of the hemisphere = side of the cube $= l$

radius of the hemisphere $= \frac{l}{2}$

T.S.A of the remaining solid = surface area of the cubical part + C.S.A of the hemispherical part − Area of the base of the hemispherical part

$$= 6 \times (\text{Edge})^2 + 2\pi r^2 - \pi r^2$$

$$= 6 \times (\text{Edge})^2 + \pi r^2$$

$$= 6 \times l^2 + \pi \left(\frac{l}{2}\right)^2 = \frac{l^2}{4}\pi + 6l^2$$

$$= \frac{1}{4} l^2 (24 + \pi)$$

total surface area of the remaining solid

$$= \frac{1}{4}(24 + \pi)\, l^2 \text{ sq. units.}$$

Example 7.29

A metallic sphere of radius 16cm is melted and recast into small spheres each of radius 2cm. How many small spheres can be obtained?

Sol

Let the number of small sphere obtained be n

Let r be the radius of each small sphere and R be the radius of metallic sphere.

Here, $R = 16cm$, $r = 2cm$

n × (volume of a small sphere) = volume of big metallic sphere

$$n\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$$

$$n\left(\frac{4}{3}\pi \times 2^3\right) = \frac{4}{3}\pi \times (16)^3$$

$$n8 = (16)^3$$

$$8n = 4096$$

$$n = 4096/8$$

$$n = 512$$

∴ there will be 512 small spheres.

Example 7.30

A cone of height 24cm is made up of modeling clay. A child reshapes it in the form of a cylinder of same radius as cone. Find the height of the cylinder.

Sol

Let h_1 and h_2 be the heights of a cone and cylinder respectively.

Also, let r be the radius of the cone.

Given:

height of the cone $h_1 = 24cm$

Radius of the cone and cylinder $r = 6cm$

volume of cylinder = volume of cone

$$\pi r^2 h_2 = \frac{1}{3}\pi r^2 h_1$$

$h_2 = \frac{1}{3} h_1$

$h_2 = \frac{1}{3} \times 24 = 8$

$\therefore$ Height of cylinder is 8cm.

Example 7.31

A right circular cylindrical container of base radius 6cm and height 15cm is full of ice cream. The ice cream is to be filled in cones of height 9cm and base radius 3cm, having a hemispherical cap. Find the number of cones needed to empty the container.

Sol Let h and r be the height and radius of the cylinder respectively.

Given: $h = 15cm$, $r = 6cm$

Volume of container $V = \pi r^2 h$ cubic units.

$= \frac{22}{7} \times 6 \times 6 \times 15$

Let $r_1 = 3cm$, $h_1 = 9cm$ be the radius and height of the cone.

Also, $r_1 = 3cm$ is the radius of the hemispherical cap.

Volume of one ice cream cone = Volume of the cone + Volume of the hemispherical cap.

$= \frac{1}{3} \pi r_1^2 h_1 + \frac{2}{3} \pi r_1^3$

$= \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 9 + \frac{2}{3} \times \frac{22}{7} \times 3 \times 3 \times 3$

$= \frac{22}{7} \times 9 (3+2) = \frac{22}{7} \times 45$

$$\text{Number of cones} = \frac{\text{volume of cylinder}}{\text{Volume of one ice cream cone}}$$

$$\left.\begin{array}{l}\text{Number of ice cream}\\ \text{cones needed}\end{array}\right\} = \frac{\frac{22}{7} \times 6 \times 6 \times 15}{\frac{22}{7} \times 45} = 12$$

Thus 12 ice cream cones are required to empty the cylindrical container.

Xth HAND WRITTEN MATERIALS

CHAPTER – 8 (Examples)

STATISTICS AND PROBABILITY

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CHAPTER - 8

STATISTICS AND PROBABILITY

Example 8.1

Find the range and coefficient of Range of the following data: 25, 67, 48, 53, 18, 39, 44.

Sol Largest value $L = 67$, Smallest value $S = 18$

$$\text{Range} = L - S = 67 - 18 = 49$$

$$\text{Coefficient of Range} = \frac{L-S}{L+S} = \frac{67-18}{67+18} = \frac{49}{85} = 0.576$$

Example 8.2

Find the range of the following distribution

Age (in years)	16 - 18	18 - 20	20 - 22	22 - 24	24 - 26	26 - 28
Number of Students	0	4	6	8	2	2

Sol

Largest value $L = 28$

Smallest value $S = 18$

Range $R = L - S = 28 - 18 = 10$ years.

Example 8.3

The range of a set of data is 13.67 and the largest value is 70.08. Find the smallest value.

Sol

Range $R = 13.67$

Largest value $L = 70.08$

$$\text{Range } R = L - S$$

$$13.67 = 70.08 - S$$

$$S = 70.08 - 13.67$$

$$S = 56.41$$

$\therefore$ The smallest value is 56.41

Example 8.4

The number of television sold in each day of a week are 13, 8, 4, 9, 7, 12, 10. Find its standard deviation.

Sol

x_i	x_i^2
13	169
8	64
4	16
9	81
7	49
12	144
10	100
$\Sigma x_i = 63$	$\Sigma x_i^2 = 623$

standard deviation

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$= \sqrt{\frac{623}{7} - \left(\frac{63}{7}\right)^2}$$

$$= \sqrt{\frac{623}{7} - \frac{3969}{49}}$$

$$= \sqrt{89 - 81} = \sqrt{8}$$

Hence, $\sigma \simeq 2.83$.

Example 8.5

The amount of raintall in a particular season for 6 days are given as 17.8cm, 19.9cm, 16.3cm, 12.5cm, 12.8cm and 11.4cm. Find its standard deviation.

Sol

Arranging the numbers in ascending order we get 11.4, 12.5, 12.8, 16.3, 17.8, 19.2. Number of observation $n = 6$ (223)

$$\text{Mean} = \frac{11.4 + 12.5 + 12.8 + 16.3 + 17.8 + 19.2}{6} = \frac{90}{6} = 15$$

x_i	$d_i = x_i - \bar{x}$ $= x - 15$	d_i^2
11.4	-3.6	12.96
12.5	-2.5	6.25
12.8	-2.2	4.84
16.3	1.3	1.69
17.8	2.8	7.84
19.2	4.2	17.64
		$\Sigma d_i^2 = 51.22$

$$\text{Standard deviation } \sigma = \sqrt{\frac{\Sigma d_i^2}{n}}$$

$$= \sqrt{\frac{51.22}{6}} = \sqrt{8.53}$$

Hence, $\sigma \simeq 2.9$

Example 8.6

The marks scored by 10 students in a class test are 25, 29, 30, 33, 35, 37, 38, 40, 44, 48. Find the standard deviation.

Sol The mean of marks is 35.9

$$\bar{x} = \frac{25 + 29 + 30 + 33 + 35 + 37 + 38 + 40 + 44 + 48}{10} = 35.9$$

which is not integer. We take assumed mean $A = 35$, $n = 10$.

x_i	$d_i = x_i - A$ $d_i = x_i - 35$	d_i^2
25	-10	100
29	-6	36
30	-5	25
33	-2	4
35	0	0
37	2	4
38	3	9
40	5	25
44	9	81
48	13	169
	$\Sigma d_i = 9$	$\Sigma d_i^2 = 453$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2}$$

$$= \sqrt{\frac{453}{10} - \left(\frac{9}{10}\right)^2}$$

$$= \sqrt{\frac{453}{10} - \frac{81}{100}}$$

$$= \sqrt{45.3 - 0.81} = \sqrt{44.49}$$

$$\sigma \simeq 6.67$$

Example 8.7

The amount that the children have spent for purchasing some eatables in one day trip of a school are 5, 10, 15, 20, 25, 30, 35, 40. Using step deviation method to find the standard deviation of the amount they have spent.

Sol

All the observations are divisible by 5. Hence we can use the step deviation method.

Let the Assumed Mean $A = 20, n = 8$.

x_i	$d_i = x_i - A$ $d_i = x_i - 20$	$d_i = \frac{x_i - A}{c}$ $c = 5$	d_i^2
5	-15	-3	9
10	-10	-2	4
15	-5	-1	1
20	0	0	0
25	5	1	1
30	10	2	4
35	15	3	9
40	20	4	16
		$\Sigma d_i = 4$	$\Sigma d_i^2 = 44$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2} \times c$$

$$= \sqrt{\frac{44}{8} - \left(\frac{4}{8}\right)^2} \times 5 = \sqrt{\frac{11}{2} - \frac{1}{4}} \times 5$$

$$= \sqrt{5.5 - 0.25} \times 5 = \sqrt{5.25} \times 5$$

$$= 2.29 \times 5$$

$$\sigma \simeq 11.45$$

Example 8.8

Find the standard deviation of the following data 7, 4, 8, 10, 11. Add 3 to all the values then find the standard deviation for the new values.

Sol

Arranging the values in ascending order

4, 7, 8, 10, 11 and $n = 5$

x_i	x_i^2
4	16
7	49
8	64
10	100
11	121
$\Sigma x_i = 40$	$\Sigma x_i = 350$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$= \sqrt{\frac{350}{5} - \left(\frac{40}{5}\right)^2}$$

$$= \sqrt{\frac{350}{5} - \frac{1600}{25}}$$

$$\sigma = \sqrt{6} \simeq 2.45$$

When we add 3 to all the values, we get the new values as 7, 10, 11, 13, 14

x_i	x_i^2
7	49
10	100
11	121
13	169
14	196
$\Sigma x_i = 55$	$\Sigma x_i^2 = 635$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$\sigma = \sqrt{\frac{635}{5} - \left(\frac{55}{5}\right)^2}$$

$$\sigma = \sqrt{127 - 121} = \sqrt{6}$$

$$\therefore \sigma \simeq 2.45$$

Example 8.9

Find the standard deviation of the data 2, 3, 5, 7, 8. Multiply each data by 4. Find the standard deviation of the new values.

Sol Given, $n = 5$

x_i	x_i^2
2	4
3	9
5	25
7	49
8	64
$\Sigma x_i = 25$	$\Sigma x_i^2 = 151$

Standard deviation $\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

$$\sigma = \sqrt{\frac{151}{5} - \left(\frac{25}{5}\right)^2}$$

$$\sigma = \sqrt{30.2 - 25} = \sqrt{5.2}$$

$$\sigma \simeq 2.28$$

Multiply each data by 4, we get the new values are 8, 12, 20, 28, 32

x_i	x_i^2
8	64
12	144
20	400
28	784
32	1024
$\Sigma x_i = 100$	$\Sigma x_i^2 = 2416$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$= \sqrt{\frac{2416}{5} - \left(\frac{100}{5}\right)^2}$$

$$= \sqrt{483.2 - 400}$$

$$= \sqrt{83.2}$$

$$\sigma = \sqrt{16 \times 5.2} = 4\sqrt{5.2}$$

$$\sigma \simeq 9.12.$$

Example 8.10

Find the mean and varience of the first n natural numbers.

Sol

$$\text{Mean } \bar{x} = \frac{\text{sum of all the observations}}{\text{Number of observations}}$$

$$= \frac{\Sigma x_i}{n} = \frac{1+2+3+\cdots+n}{n} = \frac{n(n+1)}{2 \times n}$$

$$\text{Mean } \bar{x} = \frac{n+1}{2}$$

$$\text{Varience } \sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2 \quad \left[\begin{array}{l} \Sigma x_i^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 \\ (\Sigma x_i)^2 = (1+2+3+\cdots+n)^2 \end{array}\right]$$

$$= \frac{n(n+1)(2n+1)}{6\times n} - \left[\frac{n(n+1)}{2\times n}\right]^2$$

$$= \frac{2n^2+3n+1}{6} - \frac{n^2+2n+1}{4}$$

Varience $\sigma^2 = \frac{4n^2+6n+2-3n^2-6n-3}{12} = \frac{n^2-1}{12}$

Example 8.11

48 students were asked to write the total number of hours per week they spent on watching television. With this information find the standard deviation of hours spent for watching television.

x	6	7	8	9	10	11	12
f	3	6	9	13	8	5	4

Sol

x_i	f_i	$x_i f_i$	$d_i = x_i - \bar{x}$	d_i^2	$f_i d_i^2$
6	3	18	-3	9	27
7	6	42	-2	4	24
8	9	72	-1	1	9
9	13	117	0	0	0
10	8	80	1	1	8
11	5	55	2	4	20
12	4	48	3	9	36
	$N=48$	$\Sigma x_i f_i = 432$	$\Sigma d_i = 0$		$\Sigma f_i d_i^2 = 124$

Mean:

$$\bar{x} = \frac{\Sigma x_i f_i}{N} = \frac{432}{48} = 9 \quad (\because N = \Sigma f_i)$$

Standard deviation :

$$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N}} = \sqrt{\frac{124}{48}} = \sqrt{2.58}$$

$$\sigma \simeq 1.6$$

Example 8.12

The marks scored by the students in a slip test are given below. Find the standard deviation of their marks.

x	4	6	8	10	12
f	7	3	5	9	5

Sol

Assumed mean $A = 8$

x_i	f_i	$d_i = x_i - A$	$f_i d_i$	$f_i d_i^2$
4	7	-4	-28	112
6	3	-2	-6	12
8	5	0	0	0
10	9	2	18	36
12	5	4	20	80
	$N = 29$		$\Sigma f_i d_i = 4$	$\Sigma f_i d_i^2 = 240$

Standard deviation :

$$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$$

$$= \sqrt{\frac{240}{29} - \left(\frac{4}{29}\right)^2} = \sqrt{\frac{240 \times 29 - 16}{29 \times 29}}$$

$$\sigma = \sqrt{\frac{6944}{29 \times 29}} = \sqrt{\frac{6944}{841}}$$

$$\sigma = \sqrt{8.26}$$

$$\sigma \simeq 2.87$$

Example 8.13

Marks of the students in a particular subject of a class are given below. Find its standard deviation.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of students	8	12	17	14	9	7	4

Sol

Let Assume mean $A = 35$, $c = 10$

Marks	Mid value (x_i)	f_i	$d_i = x_i - A$	$d_i = \frac{x_i - A}{c}$	$f_i d_i$	$f_i d_i^2$
0-10	5	8	-30	-3	-24	72
10-20	15	12	-20	-2	-24	48
20-30	25	17	-10	-1	-17	17
30-40	35	14	0	0	0	0
40-50	45	9	10	1	9	9
50-60	55	7	20	2	14	28
60-70	65	4	30	3	12	36
		$N = 71$			$\Sigma f_i d_i = -30$	$\Sigma f_i d_i^2 = 210$

Standard deviation $\sigma = c \times \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

$$\sigma = 10 \times \sqrt{\frac{210}{71} - \left(\frac{30}{71}\right)^2}$$

$$\sigma = 10 \times \sqrt{\frac{210}{71} - \frac{900}{5041}}$$

$$\sigma = 10 \times \sqrt{2.779} \Rightarrow 10 \times 1.667$$

$$\sigma \simeq 16.67$$

Example 8.14

The mean and standard deviation of 15 observations are found to be 10 and 5 respectively. on rechecking it was found that one of the observation with value 8 was incorrect. calculate the correct mean and standard deviation if the correct observation value was 23?

Sol

$n = 15$, $\bar{x} = 10$, $\sigma = 5$; $\bar{x} = \frac{\Sigma x}{n}$; $\Sigma x = \bar{x} \times n$

$= 15 \times 10 = 150$

Wrong observation value $= 8$

Correct observation value $= 23$.

Correct value $= 150 - 8 + 23 = 165$

Correct mean $\bar{x} = \frac{165}{15} = 11$

standard deviation $\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

Incorrect value of $\sigma = 5 = \sqrt{\frac{\Sigma x^2}{15} - (10)^2}$

$$25 = \frac{\Sigma x^2}{15} - 100$$

$$\frac{\Sigma x^2}{15} = 100 + 25$$

$$\frac{\Sigma x^2}{15} = 125$$

Incorrect value of $\Sigma x^2 = 1875$

Correct value of $\Sigma x^2 = 1875 - 8^2 + 23^2$

$= 2340$

Corrected standard deviation

$$\sigma = \sqrt{\frac{2340}{5} - (11)^2}$$

$$\sigma = \sqrt{156 - 121}$$

$$\sigma = \sqrt{35}$$

$$\sigma \simeq 5.9$$

Example 8.15

The mean of a data is 25.6 and its coefficient of variation is 18.75. Find the standard deviation.

Sol

Mean $\bar{x} = 25.6$

Coefficient of variation $C.V = 18.75$

Coefficient of variation $C.V = \frac{\sigma}{\bar{x}} \times 100\%$

$$18.75 = \frac{\sigma}{25.6} \times 100$$

$$\sigma = \frac{18.75 \times 25.6}{100}$$

$$\sigma = \frac{480}{100}$$

$$\sigma = 4.8$$

Example 8.16

The following table gives the values of mean and variance of heights and weights of the 10th standard students of a school.

	Height	weight
Mean	155cm	46.50 kg
varience	72.25cm²	28.09 kg

which is more varying than the other?

30)

For comparing two data, first we have to find their coefficient of variations.

Mean $\bar{x}_1 = 155cm$, varience $\sigma_1^2 = 72.25cm^2$

Standard deviation $\sigma_1 = 8.5$

Coefficient of variation $C.V_1 = \frac{\sigma_1}{x_1} \times 100\%$

$$C.V_1 = \frac{8.5}{155} \times 100\%$$

$C.V_1 = 5.48\%$ (for heights)

Mean $\bar{x}_2 = 46.50 kg$, variance $\sigma_2^2 = 28.09 kg^2$

Standard deviation $\sigma_2 = 5.3 kg$.

Coefficient of variation $C.V_2 = \frac{\sigma_2}{\bar{x}_2} \times 100\%$

$$C.V_2 = \frac{5.3}{46.50} \times 100\%$$

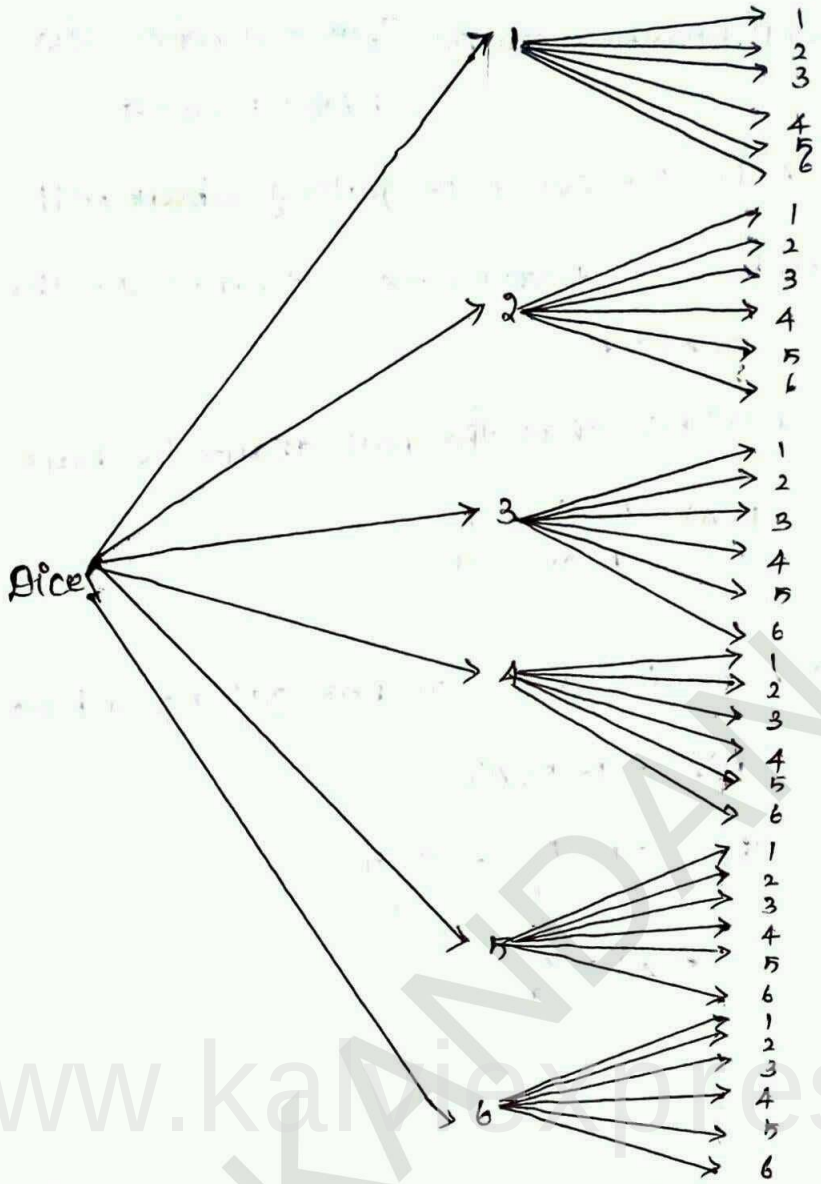
$C.V_2 = 11.40\%$ (for weights)

$C.V_1 = 5.48\%$ and $C.V_2 = 11.40\%$

Height is more consistent.

Example 8.17

Express the sample space for rolling two dice using tree diagram.



The sample space can be written as,

$S = \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \}$

Example 8.18

A bag contains 5 blue balls and 4 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is (i) blue (ii) not blue.

Sol

Total number of possible outcomes $n(S)$

$n(S) = 5 + 4 = 9$

(i) Let A be the event of getting a blue ball

Number of favourable outcomes for the event A. $n(A) = 5$

Probability that the ball drawn is blue.

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{9}$$

(ii) Let $\bar{A}$ be the event of not getting a blue ball

$$P(\bar{A}) = 1 - P(A)$$

$$= 1 - \frac{5}{9} = \frac{9-5}{9}$$

$$P(\bar{A}) = \frac{4}{9}$$

Example 8.19

Two dices are rolled. Find the probability that the sum of outcomes is (i) equal to 4 (ii) greater than 10 (iii) less than 13.

Sol

When we roll two dice, the sample space is given by

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

$n(S) = 36$

(i) Let A be the event of getting the sum of outcomes values equal to 4.

then, $A = \{(1,3), (2,4), (3,1)\}$

$n(A) = 3$

Probability of getting the sum of outcomes equal to A is

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{3}{36} = \frac{1}{12}$$

(ii) Let B be the event of getting the sum of outcomes values greater than 10

$B = \{(5,6), (6,6), (6,5)\}$

$n(B) = 3$

Probability of getting the sum of outcomes values greater than 10 is

$$P(B) = \frac{n(B)}{n(S)}$$

$$P(B) = \frac{3}{36} = \frac{1}{12}$$

(iii) Let C be the event of getting the sum of outcomes less than 13. Here All the outcomes have the sum value less than 13. Hence $C = S$

$n(C) = n(S) = 36$

Probability of getting the total value less than 13 is

$$P(C) = \frac{n(C)}{n(S)} = \frac{36}{36} = 1$$

Example 8.20.

Two coins are tossed together. What is the probability of getting different faces on the coins?

Sol When two coins are tossed together, the sample space is $S = \{HH, HT, TH, TT\}$; $n(S) = 4$

Let A be the event of getting different faces on the coins.

$$A = \{HT, TH\} \; ; \; n(A) = 2$$

Probability of getting different faces on the coins is

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

Example 8.21

From a well shuffled pack of 52 cards, one card is drawn at random. Find the probability of getting (i) Red card (ii) heart card (iii) Red King (iv) face card (v) number card.

Sol

$$n(S) = 52$$

(i) Let A be the event of getting a red card

$$n(A) = 26$$

Probability of getting a red card is

$$P(A) = \frac{n(A)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

(ii) Let B be the event of getting a heart card

$$n(B) = 13$$

Probability of getting a heart card is

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

(iii) Let C be the event of getting a red King card can be either a diamond King or a heart King.

$$n(C) = 2$$

Probability of getting a red King card is

$$P(C) = \frac{n(C)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

(iv) Let D be the event of getting a face card. the face cards are Jack (J), Queen (Q), King (K).

$$n(D) = 4 \times 3 = 12$$

(238)

Probability of getting a face card is

$$P(D) = \frac{n(D)}{n(S)} = \frac{12}{52} = \frac{3}{13}$$

(v) Let E be the event of getting a number card. the Number cards are. 2, 3, 4, 5, 6, 7, 8, 9 and 10

$$n(E) = 4 \times 9 = 36$$

Probability of getting a number card is

$$P(E) = \frac{n(E)}{n(S)} = \frac{36}{52} = \frac{9}{13}$$

Example 8.22

What is the probability that a leap year selected at random will contain 53 saturdays.

Sol

A leap year has 366 days. So it has 52 full weeks and 2 days. 52 saturdays must be in 52 full weeks.

The possible change of remaining two days will be the sample space.

S = { Sun - Mon, Mon - Tue, Tue - wed, wed - thu, thu - fri, fri - sat, sat - Sun }

$n(S) = 7$

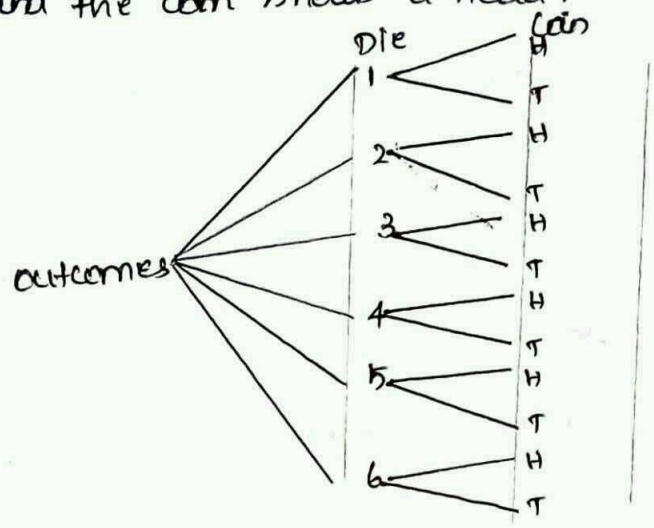
Let A be the event getting 53rd saturday.

A = { Fri - Sat, sat - Sun } ; $n(A) = 2$

Probability of getting 53 saturdays in a leap year is $P(A) = \frac{n(A)}{n(S)} = \frac{2}{7}$

Example 8.23

A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head.



Sol

Sample space

$S = \{1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T\}$

$n(S) = 12$

Let A be the event of getting an odd number and a head.

$A = \{1H, 3H, 5H\}$; $n(A) = 3$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{12} = \frac{1}{4}$$

Example 8.24

A Bag contains 6 green balls, some black and red balls. Number of black balls is as twice as the number of red balls. probability of getting a green Ball is thrice the probability of getting a red ball. Find (i) Number of black balls (ii) total number of balls

Sol

Number of green balls is $n(G) = 6$

Number of red balls is $n(R) = x$

Number of Black balls is $n(B) = 2x$

Total number of balls $n(S) = 6 + x + 2x$

$= 6 + 3x$

Given, $P(G) = 3 \times P(R)$

$$\frac{6}{6+3x} = 3 \times \frac{x}{6+3x}$$

$$6 = 3x$$

$$x = 6/3$$

$$\boxed{x = 2}$$

(i) Number of black balls $= 2 \times 2 = 4$

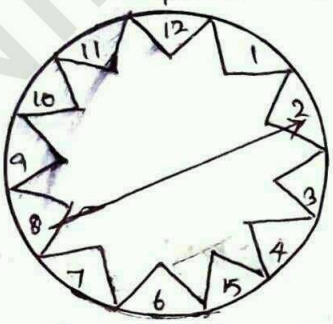
(ii) Total number of balls $= 6 + (3 \times 2)$

$= 6 + 6$

$= 12$

Example 8.25

A game of chance consists of spinning an arrow which is equally likely to come to rest pointing to one of the numbers 1, 2, 3, ..., 12. What is the probability that it will point to (i) 7 (ii) a prime number (iii) a composite number?



Sol

Sample Space $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$n(S) = 12$

(i) Let A be the event of resting in 7, $n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{12}$$

(ii) Let B be the event that the arrow will come to rest in a prime number.

$B = \{2, 3, 5, 7, 11\}$

$n(B) = 5$

$P(B) = \frac{n(B)}{n(S)} = \frac{5}{12}.$

(iii) Let C be the event that arrow will come to rest in a composite number.

$C = \{4, 6, 8, 9, 10, 12\}$

$n(C) = 6$

$P(C) = \frac{n(C)}{n(S)} = \frac{6}{12} = \frac{1}{2}$

Example 8.26

If $P(A) = 0.37$, $P(B) = 0.42$, $P(A \cap B) = 0.09$, then find $P(A \cup B)$

Sol

$P(A) = 0.37$, $P(B) = 0.42$, $P(A \cap B) = 0.09$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.37 + 0.42 - 0.09$

$P(A \cup B) = 0.79 - 0.09$

$P(A \cup B) = 0.7$

Example 8.27

What is the probability of drawing either a king or a queen in a single draw from a well shuffled pack of 52 cards?

Sol

Total number of cards = 52

Number of king cards = 4

Probability of drawing a king card $= \frac{4}{52}$

Number of queen cards = 4

Probability of drawing a queen card $= \frac{4}{52}$

Both the event of drawing a king and a queen are mutually exclusive. (342)

$$P(A \cup B) = P(A) + P(B)$$

$\therefore$ probability of drawing either a king or a queen.

$$= \frac{4}{52} + \frac{4}{52}$$

$$= \frac{8}{52} = \frac{2}{13}$$

Example 8.28

Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

Sol

When two diced are rolled together, there will be $6 \times 6 = 36$ outcomes. Let S be the sample space. then $n(S) = 36$

Let A be the event of getting a doublet and B be the event of getting face sum 4.

$$A = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$$

$$B = \{(1,3), (2,2), (3,1)\}$$

$$\therefore A \cap B = \{(2,2)\}$$

then, $n(A) = 6$, $n(B) = 3$, $n(A \cap B) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{36}$$

$\therefore$ P(getting a doublet or a total of 4) = $P(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$ (943)

$$= \frac{6}{36} + \frac{3}{36} - \frac{1}{36}$$

$$= \frac{6+3-1}{36} = \frac{8}{36} = \frac{2}{9}$$

Hence, the required probability is $\frac{2}{9}$

Example 8.29

If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \text{ and } B) = \frac{1}{8}$, find (i) P(A or B) (ii) P(not A and not B).

Sol

(i) $P(A \text{ or } B) = P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B)$$

$$P(A \text{ or } B) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8}$$

$$= \frac{2+4-1}{8} = \frac{5}{8}$$

(ii) $P(\text{not } A \text{ and not } B) = P(\bar{A} \cap \bar{B})$

$$= P(\overline{A \cup B})$$

$$= 1 - P(A \cup B)$$

$$P(\text{not } A \text{ and not } B) = 1 - \frac{5}{8} = \frac{3}{8}$$

Example 8.30

A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card.

Sol

Total number of cards = 52

$n(S) = 52$

Let A be the event of getting a king card.

$n(A) = 4$

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52}$

Let B be the event of getting a heart card

$n(B) = 13$

$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52}$

Let C be the event of getting a red card

$n(C) = 26$

$P(C) = \frac{n(C)}{n(S)} = \frac{26}{52}$

$P(A \cap B) = P(\text{getting heart king}) = \frac{1}{52}$

$P(B \cap C) = P(\text{getting red and heart}) = \frac{13}{52}$

$P(A \cap C) = P(\text{getting red king}) = \frac{2}{52}$

$P(A \cap B \cap C) = P(\text{getting heart, king which is red})$

$= \frac{1}{52}$

Required probability is

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - \frac{13}{52} - \frac{2}{52} + \frac{1}{52}$

$= \frac{4 + 13 + 26 - 1 - 13 - 2 + 1}{52} = \frac{28}{52}$

$= \frac{7}{13}$

Example 8.31

In a class of 50 students, 28 opted for NCC, 30 opted for NSS and 18 opted both NCC and NSS. One of the students is selected at random. Find the probability that.

(i) The student opted for NCC and NSS

(ii) the student opted for NSS but Not NCC

(iii) The student opted for exactly one of them.

Sol total numbers of students $n(S) = 50$,

Let A and B be the events of students opted for NCC and NSS respectively.

$n(A) = 28$, $n(B) = 30$, $n(A \cap B) = 18$

$$P(A) = \frac{n(A)}{n(S)} = \frac{28}{50}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{30}{50}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{18}{50}$$

(i) probability of the students opted for NCC but not NSS.

$$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

$$= \frac{28}{50} - \frac{18}{50} = \frac{10}{50} = \frac{1}{5}$$

(ii) probability of the students opted for NSS but not NCC.

$$P(\bar{A} \cap B) = P(B) - P(A \cap B)$$

$$= \frac{30}{50} - \frac{18}{50} = \frac{30-18}{50} = \frac{12}{50} = \frac{6}{25}$$

(iii) probability of the students opted for exactly one of them.

$$= P[(A \cap \bar{B}) \cup (\bar{A} \cap B)]$$

$$= P(A \cap \bar{B}) + P(\bar{A} \cap B)$$

$$= \frac{1}{5} + \frac{6}{25}$$

$$= \frac{5+6}{25} = \frac{11}{25}$$

Example 8.32.

A and B are two candidates seeking admission to IIT. the probability that A getting selected is 0.5 and the probability that both A and B getting selected is 0.3. prove that probability of B being selected is atmost 0.8.

Sol

$$P(A) = 0.5, \quad P(A \cap B) = 0.3$$

$$P(A \cup B) \leq 1$$

$$P(A) + P(B) - P(A \cap B) \leq 1$$

$$0.5 + P(B) - 0.3 \leq 1$$

$$P(B) \leq 1 - 0.2$$

$$P(B) \leq 0.8$$

$\therefore$ probability of B getting selected is atmost 0.8.
